MATH 231A: LECTURE 3 REFERENCES

1. Mean Value Property and Strong Maximum Principle

- This material is explained well in Evans Chapter 2.2.2-2.2.3. For another version of the calculations in a different proof in 2D using Poisson's expansion, see Strauss Chapter 6.3. Strauss also contains a proof of the 3D MVP in Chapter 7.1 using spherical coordinates. For yet another rendition, refer to Chapter 1 of Jost.

- There are a variety of maximum principles for the Laplace equation, as well as second order "elliptic" equations of the form $L = \sum_{i,j} a^{ij}(x)\partial_i\partial_j$ where a^{ij} is a positive definite matrix valued function. What is interesting is that these can be derived from the equation more directly *without* using the MVP (which is not even true for general elliptic equations). For more information, see Chapter 2 of Jost.

2. The Dirichlet Principle

- The basic calculation performed in class are in Section 2.2.5 of Evans.

- For a very nice account of the Dirichlet principle in the context of classical analysis, refer to the classical book of Richard Courant [1]. For more on the history of Courant's contribution to the subject (which was somewhat controversial!), read the seminal biography of Constance Reid [2].

References

[1] Courant, Richard **Dirichlet's principle, conformal mapping, and minimal surfaces. With an appendix by M. Schiffer.** Reprint of the 1950 original. Springer-Verlag, New York-Heidelberg, 1977. xi+332 pp.

[2] Reid, Constance **Courant in Göttingen and New York. The story of an improbable mathematician.** Springer-Verlag, New York-Heidelberg, 1976. ii+314 pp.