2009 Advanced Calculus 142A – Homework 4

1. For each of the following sequences, either prove that it converges (and find the limit) or prove that it diverges (and if it diverges to $\pm\infty$, prove that fact too):

(i).
$$x_n = \frac{n-1}{2n+1}$$

(ii).
$$x_n = \frac{4n^2+3}{n-4}$$

(iii).
$$x_n = \frac{n^3-2}{n^4-3}$$

(iv).
$$x_n = (1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})\cdots(1-\frac{1}{n})$$

2. If (x_n) is a sequence which converges to a positive number x, show that the sequence $(\sqrt{x_n})$ converges to $\sqrt{x}$. Hint:
$$\frac{a-b}{\sqrt{a}+\sqrt{b}} = \sqrt{a}-\sqrt{b}$$

3. For each of the following sequences, either prove that it converges (and find the limit) or prove that it diverges (and if it diverges to $\pm\infty$, prove that fact too):

(i). $\sqrt{n+1}-\sqrt{n}$

(ii). $\sqrt{2n+1}-\sqrt{n}$

4. Suppose (x_n) and (y_n) are sequences which both diverge to $+\infty$. Show that (x_n+y_n) must also diverge to $+\infty$, but give examples to show that (x_n-y_n) can go to a finite limit, $\pm\infty$, or just diverge (meaning, do none of these).

5. Suppose (x_n) is a sequence of positive numbers which diverges to $+\infty$. Show that $(\sqrt{x_n})$ is a sequence which also diverges to $+\infty$.