Vector Calculus 20E, Fall 2014, Lecture A, Midterm 2

Fifty minutes, three problems. No calculators allowed.
Please start each problem on a new page.
You will get full credit only if you show all your work clearly.
Simplify answers if you can, but don’t worry if you can’t!

1. Let Σ be the part of the paraboloid \( z = x^2 + y^2 \) lying between \( z = 1 \) and \( z = 4 \). Compute the surface area of Σ.

2. Let \( C \) be a circle of radius 1, lying in the plane \( 2x + 2y + z = 5 \), and centred at the point \((1, 1, 1)\). Orient \( C \) clockwise, as seen from the origin. Let \( \mathbf{F} \) be the vector field given by \( \mathbf{F}(x, y, z) = (3 \cos x + z, 5x - e^y, z^4 - 3y) \). Compute the circulation \( \int_C \mathbf{F}.d\mathbf{s} \).

3. Let Σ be the part of the hyperboloid \( x^2 + y^2 = 1 + z^2 \) lying between \( z = -1 \) and \( z = +1 \), oriented using normals pointing outward, away from the z-axis. Let \( \mathbf{F} \) be the vector field given by \( \mathbf{F}(x, y, z) = (x, y, z) \). Compute the flux \( \int_\Sigma \mathbf{F}.d\mathbf{S} \)

If you have any comments for me about the course so far (things you like, dislike, would like, etc.), please write them in the space below and hand this paper in at the end of the exam. You’ll remain anonymous – I can’t identify your handwriting!

(If you want to keep the questions, tear off the top part of this paper, but do it quietly!)