1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (xy, x + y^2)$. Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be the composite function $g = f \circ f$, so that $g(x, y) = f(f(x, y))$. Compute the matrix of the derivative $Dg$ at the point $(1, 2)$.

2. Compute the second-order Taylor approximation, at the point $(1, \pi/2)$, to the function $f(x, y) = \sin(x^2y)$: express your answer in the form

\[ f(1 + h_1, \pi/2 + h_2) = \ldots \]

3. Let $D$ be the region in $\mathbb{R}^2$ where $x \geq 0, y \geq 0$ and $x^{1/3} + y^{1/3} \leq 1$. Use the change of variables $x = u^3, y = v^3$ to calculate the area of $D$.

If you have any comments for me about the course so far (things you like, dislike, would like, etc.), please write them in the space above, tear it off and hand it in at the end of the exam. (I don’t know what your writing looks like, so you’ll remain anonymous!)