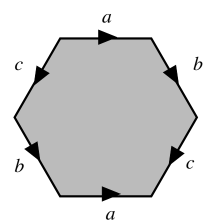
8. Cellular homology

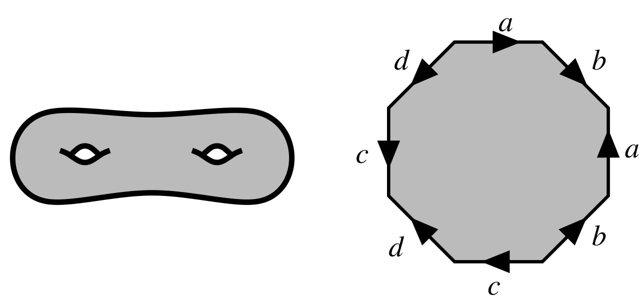
Use cellular chain complexes to compute the homology of the following spaces

1. $S^m \vee S^n$ and $S^m \times S^n$.

2. The space obtained by identifying the edges of a solid hexagon as shown below.



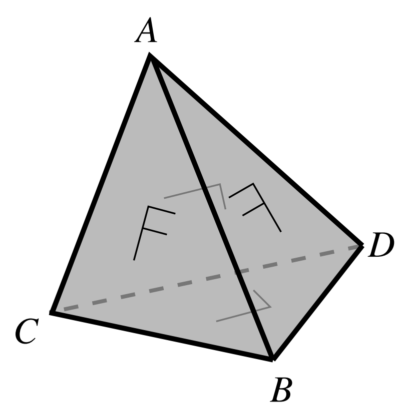
3. The orientable surface of genus g; for $g = 2$ the surface and a gluing pattern for it are shown.



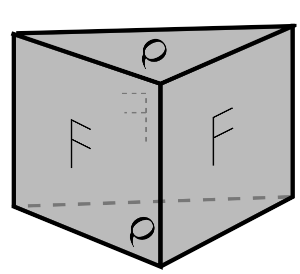
4. The non-orientable surface of genus h, made by removing h discs from a sphere and gluing Möbius strips in their places.

5. The figure-of eight space $S^1 \vee S^1$ with two discs attached along the loops $ababab$ and $ababababab$.

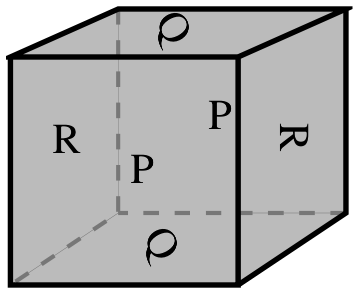
6. The space obtained by gluing up the sides of a solid tetrahedron as shown below (ABC is glued to ABD and ACD to BCD).



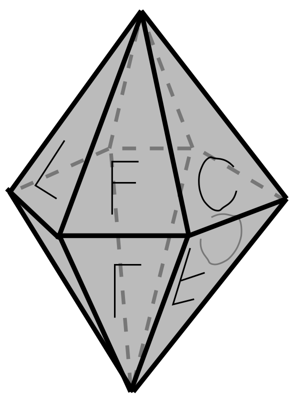
7. The solid triangular prism with its faces identified as shown:



8. The cube with faces identified in pairs according to the scheme below.



9. The *lens space* $L(p, 1)$ (for p a natural number) is defined by the following gluing construction: take a closed lens-shaped 3-ball, partition its top and bottom hemispheres into p segments, and glue these together by vertical "translation" composed with a rotation by $2\pi/p$. The picture below shows the case $p = 6$, with the lens drawn as a solid double hexagonal pyramid, and only three of the six pairs of glued faces are labelled, to avoid cluttering it up.



Describe a CW complex structure for $L(p, 1)$, and use this to compute its fundamental group and homology groups.

Degree

10. Let p be a polynomial of algebraic degree n. Viewed as a continuous self-map of the Riemann sphere $S^2 = \mathbb{C} \cup \{\infty\}$ (sending infinity to infinity), what is its topological degree?

11. Consider the rational function $p(z) = \frac{(z-a_1)(z-a_2)\cdots(z-a_n)}{(z-b_1)(z-b_2)\cdots(z-b_m)}$, where all the a_i and b_j are distinct from one another, as a function from the Riemann sphere to itself. What is its topological degree?

12. Let f be a map $S^m \to S^n$ for which there exists some open set U such that $f|_U$ is a homeomorphism to its image. Show that $m = n$ and f is surjective.