

3.3 Exercises

Question 1. Determine an expression which uniquely creates all strings in the given set of binary strings. Then find the generating function for that set of strings.

- (a) The set of strings which do not contain 111.
- (b) The set of strings none of whose blocks have length two.
- (c) The set of strings where every block of 0s has length at least two and every block of 1s has length at least three.

Question 2. Find the generating function for the set of binary strings where all blocks have the same length.

Question 3. Solve the following recurrence equations with the given initial conditions.

- (a) $a_n - 13a_{n-1} + 36a_{n-2} = 0$ where $a_0 = 0, a_1 = 5$.
- (b) $a_n - 2a_{n-1} + a_{n-2} = 0$ where $a_0 = a_1 = 1$.
- (c) $a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0$ where $a_0 = 7, a_1 = 10$ and $a_2 = 18$.

Question 4. Let S be the set of binary strings consisting of a (nonempty) block of 0s followed by a (nonempty) block of 1s, such that if the block of 0s has odd length, then the block of 1s has even length. Let a_n be the number of strings of length n in S .

- (a) Find a_0, a_1, a_2, a_3 .
- (b) Prove that

$$S = \{0\}\{0\}^*\{1\}\{1\}^* \setminus \{0\}\{00\}^*\{1\}\{11\}^*.$$

- (c) Show that the generating function for S is

$$\Phi_S(x) = \frac{x^3(2+x)}{(1-x^2)^2}.$$

- (d) Write down a recurrence equation for a_n .
- (e) Determine a closed formula for a_n for all n , and find a_{1001} .

Question 5. Write down recurrence relations with initial conditions for the quantities a_n whose generating functions $\Phi(x) = \sum_{n=0}^{\infty} a_n x^n$ are given below. Determine the asymptotic behavior of a_n in each case.

- (a) $\Phi(x) = \frac{1}{1-2x}$.
- (b) $\Phi(x) = \frac{1}{1-x-x^3}$.
- (c) $\Phi(x) = \frac{x^2+x+1}{x^3-3x^2+2}$.
- (d) $\Phi(x) = \frac{x^k-1}{x-1}$.
- (e)*s $\Phi(x) = (1-x)^{1/2}$.