

Practice Midterm 1 – Math 154

Time: 40 Minutes | No notes allowed

Questions carry equal weight | Calculators allowed

Question 1. State the Binomial Theorem, and use it to prove

$$(1 + x + x^2)^n = \sum_{k=0}^n \sum_{j=0}^k \binom{n}{k} \binom{k}{j} x^{j+k}.$$

Question 2. Prove by induction that

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

Question 3. Let $n \in \mathbb{N}$. A composition of n with k parts is a sequence (x_1, x_2, \dots, x_k) of elements of $[n]$ such that $x_1 + x_2 + \dots + x_k = n$. By convention, for $k = 0$ there is one composition, with no parts, called the empty composition. Show that there are $\binom{n-1}{k-1}$ compositions of n with k parts for $k \geq 1$.

Question 4. Let $n \in \mathbb{N}$. Determine the average number of parts in a composition of n .

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