

Practice Midterm 1 – Math 154

Time: 40 Minutes | No notes allowed

Questions carry equal weight | Calculators allowed

Question 1. Let $k \in \mathbb{N}$. Find the number of walks of length k on the number line starting at zero and which return to the origin.

Question 2. State the inclusion-exclusion formula, and use it to prove that the number of derangements of $[n]$ is $n! \sum_{k=0}^n \frac{(-1)^k}{k!}$.

Question 3. Determine closed form expressions for either one of the formal power series below:

(a) $\sum_{k=0}^{\infty} \sum_{j=0}^k (2+x)^{-j} (2-x)^{j-k}.$

(b) $\sum_{k=0}^{\infty} \sum_{n=0}^k \binom{k}{n} 2^{-n} x^{k-n}.$

Question 4. Find the number of compositions of n in which each part is odd.

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