Practice Midterm 3 – Math 154

Time: 40 Minutes | No notes allowed

Questions carry equal weight | Calculators allowed

Question 1. Let $e \approx 2.718...$ denote the base of the natural logarithm. Prove by induction on n that

$$n! \geq n^n e^{-n}.$$

You may use the fact that

$$\left(1 - \frac{1}{n}\right)^{n-1} > \frac{1}{e}.$$

Question 2. Determine the generating function for the set S of positive integers with respect to the weight function $\omega(\sigma) = \lfloor \log_2 \sigma \rfloor$. Here $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

Question 3. Determine explicitly the value of the sum

$$\sum_{k=0}^{\infty} \sum_{j=0}^{k} \binom{1/2}{j} \binom{1/2}{k-j} 2^{2k+j}.$$

Question 4. The binary strings which start with a 1 and end with a 0 and do not contain 010 or 101 are uniquely created by the formula

$$S = \{1\}\{1\}^*(\{00\}\{0\}^*\{11\}\{1\}^*)^*\{0\}\{0\}^*.$$

(a) If a_n is the number of strings of length n in S, find a_n for $n = 0, 1, 2, 3, 4$.

(b) Write down the generating function for these strings.

(c) Determine a recurrence equation for a_n, the number of strings of length n in S.

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