This assignment carries a total of twenty points, where each question has equal weight. The assignment is due on Friday April 24th by noon.

Question 1. Determine each of the following quantities explicitly.

(a) $\binom{-\frac{1}{2}}{4}$ (b) $\binom{-\frac{1}{3}}{3}$ (c) $\sum_{k=1}^{\infty} \binom{\frac{1}{3}}{k} \left(\frac{1}{7}\right)^k$ (d) $\sum_{k=1}^{\infty} \frac{1}{k2^k}$

Question 2. Prove that the inverse of $1 + 2x + 3x^2 + 2x^3 + x^4$ is

$$\sum_{j=1}^{\infty} \sum_{i=1}^{j} (-1)^{j-1} i \binom{j}{i} x^{i+j-2}.$$

Question 3. The range of a non-empty subset of $[n]$ is the difference between the largest and smallest elements of of the set.

(a) Prove that the number of subsets of $[n]$ whose range is k is exactly $(n-k)2^{k-1}$ for $k \geq 1$ and n for $k = 0$.

(b) If S is the set of non-empty subsets of $[n]$ and the weight of a set is its range, determine the generating function $\Phi_S(x)$.

(c) Determine the average range of subsets of $[n]$.

Question 4. Determine the number of compositions of n into k parts such that the ith part is i or $i+1$.