

Practice Final Examination #1

Math 154 – Combinatorics and Graph Theory

Instructor – J. Verstraete

Allotted time – 3 hours

Answers are to be written clearly and legibly
Calculators are allowed
State clearly any theorems used without proof
Total 50 points

Question 1.**10**

- (a) State precisely the inclusion exclusion formula for the union of a set of finite sets A_1, A_2, \dots, A_n . [3]
- (b) Write down the set P of prime numbers less than $\sqrt{150}$. [2]
- (c) Determine the number of positive integers less than 150 which are divisible by some prime number in P . [3]
- (d) Determine the total number of prime numbers less than 150. [2]

Question 1...

Question 2.**12**

- (a) Let S be the set of all binary strings where each non-empty block of 0s is followed by a block of 1s of equal length. Write down all the strings of length at most three in S . [2]
- (b) Use the fact that $\{1\}^* (\{0\}\{0\}^* \{1\}\{1\}^*) \{0\}^*$ uniquely creates all binary strings to show that [2]

$$S = \{1\}^* \{01, 0011, 000111, \dots\}^*$$

- uniquely creates all binary strings in S .
- (c) Write down the generating function for the number of strings of length n in S . [2]
- (d) If a_n is the number of strings of length n in S , find a recurrence equation for a_n . [2]
- (e) Solve the recurrence equation in (d) for a_n . [4]

Question 2...

Question 3.

10

- (a) For a graph G , and non-adjacent $u, v \in V(G)$, define $\kappa(u, v)$. [2]
- (b) For a graph G , and vertices $u, v \in V(G)$, define $\lambda(u, v)$. [2]
- (c) State the vertex and edge forms of Menger's Theorem. [4]
- (d) Prove that if a graph is k -connected for $k \geq 3$, then any two vertices in the graph are contained in a cycle of even length. [2]

Question 3...

Question 4.

10

- (a) Define $\Gamma(X)$ when X is a set of vertices of a graph G .
- (b) State and prove Hall's Theorem.

[2]

[8]

Question 4...

Question 5.

8

The network in Figure 1 shows the number of phone calls which can be made between from one person to another in a group of twelve people. On each phone call, one item of information can be communicated. Determine the maximum number of items of information which can be communicated from the group of people in blue shirts to the group of people in red shirts. Show all your working.

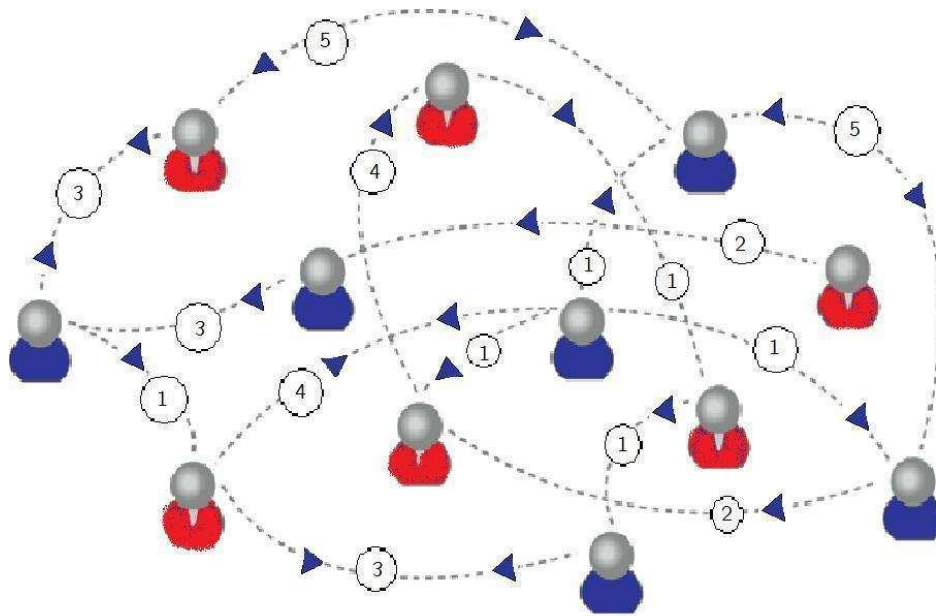


Figure 1 : Phone network

Question 5...