

Practice Final Examination #2

Math 154 – Combinatorics and Graph Theory

Instructor – J. Verstraete

Allotted time – 3 hours

Answers are to be written clearly and legibly
Calculators are allowed
State clearly any theorems used without proof
Total 50 points

Question 1.**8**

- (a) Let $d \in \mathbb{N}$. Prove that the generating function for the number of compositions of n where each part is a multiple of d is [5]

$$\Phi(x) = \frac{1 - x^d}{1 - 2x^d}.$$

- (b) Prove that there are $2^{n/d-1}$ such compositions if n is a multiple of d . [3]

Question 1...

Question 2.

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- (a) Let S be the set of all binary strings not containing 110. Write down all the strings of length 1, 2 and 3 in S . [3]
- (b) Choose the expression uniquely creates all binary strings in S
- (1) $\{0\}^* (\{1\}^* \{0\}^*)^* \{1\}^*$
- (2) $\{0\}^* (\{1\} \{0\} \{0\}^*)^* \{1\}^*$
- (3) $\{0\}^* (\{1\} \{11\}^* \{0\} \{0\}^*)^* \{1\}^*$ [2]
- (c) Write down the generating function for the number of strings of length n in S . [2]
- (d) If a_n is the number of strings of length n in S , find a recurrence equation for a_n . [2]
- (e) Find $\lim_{n \rightarrow \infty} a_n^{1/n}$. [3]

Question 2...

Question 3.

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- (a) State and prove the handshaking lemma. [3]
- (b) Prove that a tree on n vertices has $n - 1$ edges. [4]
- (c) Show that every tree has at least two vertices of degree one. [2]
- (d)* Let T and U be edge-disjoint binary trees on a set V of vertices. Prove that if $G = T \cup U$ then $\lambda(G) = \delta(G) = 2$. Is $\kappa(G) = 2$? State any theorems used without proof. [3]

Question 3...

Question 4.

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- (a) Define $\Gamma(X)$ when $X \subseteq A$ is a set of vertices of a bipartite graph $G = (A \cup B, E)$. [2]
- (b) State Hall's Theorem. [3]
- (c)* Let $n \in \mathbb{N}$. Prove that in a bipartite graph $G = (A \cup B, E)$ with $\delta(G) \geq n$ and $|A| = |B| = 2n$, there is a perfect matching using Hall's Theorem. [5]

Question 4...

Question 5...