

Practice Midterm Examination

Instructor J. Verstraete

Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

State Taylor's Theorem for a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ about zero. Define all the notation used. Then find the third order Taylor series for $\sin \cos(xy)$.

Question 2.

Use Lagrange multipliers to find the points which are closest together on the curves $y = x^2 + 1$ and $y = x$.

Question 3.

- (a) Give the precise definition of the double integral of a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ over the rectangle $R = [a, b] \times [c, d]$.
- (b) State Fubini's Theorem for the integral of a continuous function f on the rectangle $[a, b] \times [c, d]$.
- (c) Evaluate the following integral over the rectangle $R = [0, 2\pi] \times [0, 2\pi]$.

$$\int \int_R (e^{y^2} \sin x + e^{x^2} \sin y) dA.$$

Question 4.

Let $D = \{(x, y) : |x| + |y| \leq 1\}$. Prove that the following integral exists, and justify your answer.

$$\iint_D \frac{e^y}{1 - |y|} dA.$$