Practice Midterm Examination

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Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

What is the largest possible value of $w+x+y+z$ subject to the constraint $x^2+y^2+z^2+w^2 = 1$?

Question 2.

Find all extremes of the function $f(x) = x^2y^2 - x^3y - y^3x$ on the disc $x^2 + y^2 \leq 1$. Justify all your working.

Question 3.

Find the volume enclosed above the square $[0,1] \times [0,1]$ and under the surface $z = \frac{1}{(1+y)(x+y)}$. You may use the fact that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Question 4.

Evaluate

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{1-y-z} dz dy dx.$$