

Practice Midterm Examination

Instructor J. Verstraete

Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

Show that the origin is neither a local minimum nor a local maximum of the function $f(x, y) = \sin(xy)$. Use the second derivative test to determine the nature of all other critical points of the function $f(x, y) = \sin(xy)$ on \mathbb{R}^2

Question 2.

- (a) State the inverse function theorem.
- (b) Prove that the system of equations

$$\begin{aligned}x + y + z &= u \\x^2 + y^2 + z^2 &= v \\x^3 + y^3 + z^3 &= w\end{aligned}$$

can be solved for x, y, z as functions of u, v, w near any point (x, y, z) such that $(x - y)(y - z)(z - x) \neq 0$.

Question 3.

Determine $\int \int_D f dA$ when $f(x, y) = \frac{1}{x}$ and $D = \{(x, y) : 0 \leq y \leq 1, y \leq x \leq 1\}$.

Question 4.

Determine the volume of $D = \{(x, y, z) : |x| + |y| + |z| \leq 1\}$.