

Practice Midterm Examination

Instructor J. Verstraete

Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

- (a) Show that $xy \leq \frac{1}{2}(x^2 + y^2)$.
- (b) Use part (a) and the ϵ - δ definition of limits to show

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} = 0.$$

Question 2.

Define what it means for a function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ to be differentiable at a point a . Then prove that the function $f(x, y) = |xy|^{1/2}$ is not differentiable at $(x, y) = (0, 0)$.

Question 3.

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(u, v) = uv$ and let $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $g(x, y) = (y, x)$. If $\phi(x, y) = f(g(x, y))$, use the chain rule to find $\frac{\partial \phi}{\partial x}$.

Question 4.

Compute the second order Taylor formula for $f(x, y) = \log(1 + x + y)$ about the point $(x, y) = (0, 0)$.

Taylor's Theorem Not Examinable