Practice Midterm Examination

Instructor J. Verstraete

Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

State precisely the ϵ-δ definition of $\lim_{x \to a} f(x) = L$ for a function $f : \mathbb{R}^n \to \mathbb{R}$. Then prove using the ϵ-δ definition of limits that

$$\lim_{(x,y)\to(0,0)} \sin(x^2 + y^2) = 0.$$

Question 2.

Find the direction of steepest increase of the function $f(x, y) = (x+y)e^{xy}$ from the origin. What is the equation of the tangent hyperplane to the surface $z = f(x, y)$ at the origin?

Question 3.

State precisely the chain rule for determining the gradient of $\nabla(f \circ g)(a)$ where $a \in \mathbb{R}^m$ and $f : \mathbb{R}^n \to \mathbb{R}^p$ and $g : \mathbb{R}^m \to \mathbb{R}^n$ are functions. Then determine $\nabla(f \circ g)(1, 1)$ when $f : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $f(x, y) = (x, y, xy)$ and when $g : \mathbb{R} \to \mathbb{R}^2$ is defined by $g(z) = (z, 1/z)$.

Question 4.

Find all second order partial derivatives for the function $f : \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x, y, z) = (1 + x)(1 + y)(1 + z).$$