

Practice Midterm Examination

Instructor J. Verstraete

Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

Prove that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{x^2 + y^2}$$

does not exist.

Question 2.

State the definition of differentiability of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$. Find the derivatives $f_x(0,0)$ and $f_y(0,0)$ of the function $f(x,y) = x^{1/3}y^{1/3}$. State without proof whether this function is differentiable at $(0,0)$.

Question 3.

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a differentiable function where $f(x, y) = (u(x, y), v(x, y))$. Let $\phi(x, y) = f(f(x, y))$. Determine a formula for $\frac{\partial \phi}{\partial x}$ in terms of derivatives of f with respect to u and v and derivatives of u and v with respect to x and y .

Question 4.

Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a function. How many different second order partial derivatives can f have? Now suppose $f \in C^2(\mathbb{R}^3)$. How many different second order partial derivatives can f have? Find all second order partial derivatives of the function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ defined by

$$f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}.$$