

# Practice Midterm Examination

Instructor J. Verstraete

Time: 40 minutes

No notes allowed

All questions carry equal weight

## Question 1.

Define the distance  $d(x, y)$  for  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  and define the open ball  $B(a, r)$  of radius  $r$  centered at point  $a \in \mathbb{R}^n$ . Define what it means for a set  $U \subseteq \mathbb{R}^n$  to be open. Then use the definition to show the following:

- (a) Show that the set of real numbers which have a 1 in their decimal representation is an open set
- (b) Show that the set of real numbers which do not have a 1 in their decimal representation is not an open set.

## Question 2.

Prove that the following limit does not exist

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x + y^2 + z^3}.$$

**Question 3.**

Find the directional derivative of  $f(x, y, z) = xyz$  at  $(1, 1, 1)$  in the direction of the unit vector  $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ . Then find the direction in which  $f(x, y, z)$  is steepest and find the directional derivative in that direction.

**Question 4.**

Let  $f(x, y, z) = xyz$ . Determine  $\nabla f(x, y, z)$ , and then determine  $\nabla(\nabla f(x, y, z))$ .