Practice Midterm Examination

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Time: 40 minutes

No notes allowed

All questions carry equal weight

Question 1.

Prove that

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2}$$

does not exist.

If $y = mx$ then for $m \neq 0$

$$\frac{\sin xy}{x^2+y^2} = \frac{\sin mx^2}{x^2+m^2x^2}$$

$$= \frac{\sin mx^2}{mx^2} \cdot \frac{m}{1+m^2}$$

Now $\displaystyle\lim_{x\to 0} \frac{\sin mx^2}{mx^2}$

$$= \lim_{y\to 0} \frac{\sin y}{y} = 1.$$

So by product rule,

$$\lim_{x\to 0} \frac{\sin mx^2}{x^2+mx^2} = \frac{m}{1+m^2}$$

Depends on m so limit fails.

Question 2.

State the definition of differentiability of a function $f : \mathbb{R}^n \to \mathbb{R}$. Find the derivatives $f_x(0,0)$ and $f_y(0,0)$ of the function $f(x,y) = x^{1/3}y^{1/3}$. State without proof whether this function is differentiable at $(0,0)$.

See class notes

Question 3.

Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a differentiable function where $f(x, y) = (u(x, y), v(x, y))$. Let $\phi(x, y) = f(f(x, y))$. Determine a formula for $\frac{\partial \phi}{\partial x}$ in terms of derivatives of f with respect to u and v and derivatives of u and v with respect to x and y.

Too tough.

Question 4.

Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a function. How many different second order partial derivatives can f have? Now suppose $f \in C^2(\mathbb{R}^3)$. How many different second order partial derivatives can f have? Find all second order partial derivatives of the function $f : \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}.$$

In general : $3^2 = 9$ second order

f_{xx} f_{yy} f_{zz}

f_{xy} f_{xz} f_{yz}

f_{yx} f_{zy} f_{zx}

If $f \in C^2(\mathbb{R}^3)$

$f_{xy} = f_{yx}$ $\quad f_{xz} = f_{zx}$ $\quad f_{yz} = f_{zy}$

so only 6 second order.

$f_{xx} = \frac{-2z}{x^3}$ $\qquad f_{xy} = -\frac{1}{y^2} = f_{yx}$

$f_{yy} = -\frac{2x}{y^3}$ $\qquad f_{xz} = -\frac{1}{x^2} = f_{zx}$

$f_{zz} = -\frac{2y}{z^3}$ $\qquad f_{yz} = -\frac{1}{z^2} = f_{zy}$

$(x, y, z \neq 0)$ Derivatives don't exist

on planes $x=0$
$y=0$
$z=0$.