Question 1.

Prove that the statement \((p \rightarrow q) \land (q \oplus r) \land (p \land r)\) is a contradiction using truth tables or the rules of logic. Then draw a Venn diagram showing clearly why this is a contradiction.
Question 2.

Let $n$ be a positive integer. Prove that

$$
\sum_{k=1}^{n} k(n - k) = \binom{n + 1}{3}.
$$
Question 3.  

(a) Write down the number of compositions of $n$ into $k$ non-negative parts, where $n$ and $k$ are non-negative integers and $n \geq k$.

(b) State the multinomial formula, for the number of ordered partitions of an $n$ element set into parts of sizes $n_1, n_2, \ldots, n_k$. Then determine the number of ways the letters of the word BANANA can be rearranged.
Question 4. [6]

Solve the recurrence equation \( a_n = a_{n-1} + a_{n-2} \) where \( a_1 = a_2 = 1 \).
Question 5. [6]

(a) Prove that if $p$ is a prime number, and $0 < k < p$, then $p \mid \binom{p}{k}$. Find an example to show this is not true if $p$ is not prime.

(b) Prove that the product of primes greater than $n$ and less than $2n$ is less than $\binom{2n}{n}$ for any $n \geq 2$. 
Question 6.

(a) Determine $7^{30}$ modulo 31.

(b) Determine $7^{30}$ modulo 49.

(c) Determine $7^{30}$ modulo 30.
Question 7. 

(a) Find the Prüfer code for the labelled tree shown below. The root of the tree is the vertex labelled zero.
Question 7 continued...

(b) Draw the tree with Prüfer code \((0, 8, 0, 1, 8, 6, 3, 0, 3, 3, 1, 8)\).
Question 8.
Find a minimum spanning tree in the weighted graph below using Kruskal’s Algorithm starting with the shaded edge. Shade those edges which are in your minimum tree in the figure, and write down the total cost of your minimum tree.

Figure 2: A graph with edge costs

Total cost of minimum tree

[6]
Question 9

State and prove Euler’s Formula.
Question 10. [6]

(a) Determine the generating series for the set of compositions of $n$ into $k$ parts where each part of the composition is an element of the set $\{0, 2\}$.

(b) Determine the number of compositions of $n$ into $k$ parts where each part of the composition is an element of the set $\{0, 2\}$. 