Math 20D -- Midterm_1a
Solutions
Prof. Wavrik   Fall 2003

Part 1 - Short Answers (7 points each)

Match each function with a power series that represents it. Put the <u>number</u> of the correct function in the box next to each power series:

Function
1. $\sin(x)$
2. $\dfrac{1}{(1-x)^2}$
3. $\cos(x)$
4. $\dfrac{1}{(1-x^2)}$
5. $\sqrt{1-x}$

Correct number	Series
5	$1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3-\frac{5}{128}x^4\dots$
3	$1-\frac{1}{2}x^2+\frac{1}{24}x^4-\frac{1}{720}x^6\dots$
1	$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7\dots$
2	$1+2x+3x^2+4x^3+5x^4+6x^5+7x^6+8x^7\dots$
4	$1+x^2+x^4+x^6\dots$

Circle the letter of the correct answer.

6.

Suppose $\sum_{n=0}^{\infty} c_n x^n$ converges at x = 3 but its derivative $\sum_{n=1}^{\infty} n c_n x^{n-1}$ diverges at x = 3. What can be said about R, the radius of convergence of the series $\sum_{n=0}^{\infty} c_n x^n$?

a. R = -3

b. R < 3

c. R = 3

d. R > 3

e. R cannot be determined from the information given

The radius of convergence of a power series is R then it converges for |x| < R and diverges for |x| > R. The derivative and integral have the same radius of convergence as the original series. The convergence of the series at x=3 tells us that 3≤R. The divergence of the derivative at x=3 tells us 3≥R. So we have R=3.

7. Given the function $\frac{1}{4+x^2}$ which of the following is the correct power series and radius of convergence, R:

a. $\sum_{n=0}^{\infty} \frac{x^{2n}}{4^{n+1}}$ and R = ∞

b. $\sum_{n=0}^{\infty} \frac{x^{2n}}{4^{n+1}}$ and R = 2

c. $\sum_{n=0}^{\infty} \frac{x^{2n}}{4^{n+1}}$ and R = 4

d. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{4^{n+1}}$ and R = 4

e. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{4^{n+1}}$ and R = 2

f. None of the preceding

$$\frac{1}{4+x^2} = \left(\frac{1}{4}\right)\left(\frac{1}{1+\left(\frac{x}{2}\right)^2}\right)$$ Now substitute $u = -\left(\frac{x}{2}\right)^2$ in $\frac{1}{1-u} = 1+u+u^2+\ldots$

Part 2 - Written Solutions

8. State whether each of the following series is convergent or divergent.
How do you know? **You must justify your answer.** (6 points each)

a. $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ Converges by limit comparison test with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$ This converges by the alternating series.

The terms go to zero. You need to show that they go to zero monotonically. One way to do this is to show that the derivative of $f(x)=x^2/(x^3+1)$ is negative. Full credit was given if you identified the alternating series test and indicated what you needed to do.

c. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Diverges by the ratio test. See example 5 of section 11.6.

d. $\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+2}\right)^n$

Converges by the root test. See example 6 of section 11.6

e. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ Diverges. This is the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with p = ½ ≤1

Estimate the number n so that the remainder $|R_n| \le .0001 = 10^{-4}$. Your answer will be based on some method for estimating the remainder. It is not expected to be the exact n. **You must be clear about the methods you are using and how you applied them.** (10 points each)

a. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ We can use the integral estimate. $\int_{n+1}^{\infty} f(x)dx \le Rn \le \int_{n}^{\infty} f(x)dx$

In this case f(x) = x^{-2} so the integral on the right is 1/n which is $\le 10^{-4}$ and so $R_n \le 10^{-4}$ if $n \ge 10^4$

b. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$ Here we can use the alternating series estimate $|R_n| \le b_{n+1}$ We have b_{n+1} = $1/(n+1)^2$

So $n+1 \ge 100$ or $n \ge 99$ will work.