

# Groups32 - Sample Session

This is the script of a sample session with Groups32. You should go through it while running Groups32 in a separate window. Try the commands even if you have not yet learned about them.

This program is based on the stored multiplication tables for the groups of order 1 to 32. A variety of commands can be applied to these groups. You can get a list of these commands at any time by typing "H".

It is only necessary to type enough letters to distinguish a command from others – Groups32 will complete typing the name of the command for you and ask you for any additional information required.

In this sample session, what the user types is shown in red, and the remaining letters (supplied by the computer) are in blue. Everything in black is computer output.

You can get information about a command by typing "X" and then the command. The command will not execute but will give information about it.

Platform: SwiftForth ver 3.6.0  
An implementation of Forth for Windows  
(c) 1997-2016 Forth, Inc.

Groups32  
copyright 1990-2016 John J Wavrik  
Dept of Math - UCSD  
Ver 7.1.2 - March 3, 2016

Multiply permutations left->right (l) or left<-right (r) **L**

In some books, permutations are multiplied from  
left to right so  $(1\ 2)(1\ 3) = (1\ 2\ 3)$   
In other books, permutations are multiplied from  
right to left so  $(1\ 2)(1\ 3) = (1\ 3\ 2)$   
Here you choose how Groups32 is configured. In this  
sample session we use left->right.

CENTER	CENTRALIZER	CHART	CONJ-CLS
COSETS	EVALUATE	EXAMPLES	GENERATE
GROUP	HELP	IDENTIFY	INFO
ISOMORPHISM	LEFT	NORMALIZER	NUMTHEORY
ORDERS	PERMGRPS	POWERS	QUIT
RIGHT	SEARCH	STOP	SUBGROUPS
TABLE	X		

G1>>

Groups32 is built around the internal tables for  
all groups of orders 1-32 (there are 144 of them).  
A set of commands is provided to give information  
About these groups. The prompt shows which group  
is the current group.

```

G1>> CHART   Order of Groups (1-32 or 0) Number 0
Order        Groups of that order
1      1
2      2
3      3
4      4      5
5      6
6      7      8*
7      9
8     10     11    12    13* 14*
9     15     16
10    17     18*
11    19
12    20     21    22* 23* 24*
13    25
14    26     27*
15    28
16    29     30     31     32     33    34* 35* 36* 37* 38* 39* 40* 41* 42*
      Press a key to continue
17    43
18    44     45     46* 47* 48*
19    49
20    50     51     52* 53* 54*
21    55     56*
22    57     58*
23    59
24    60     61     62     63* 64* 65* 66* 67* 68* 69* 70* 71* 72* 73*
      74*
25    75     76
26    77     78*
27    79     80     81     82* 83*
28    84     85     86* 87*
29    88
30    89     90* 91* 92*
31    93
32    94     95* 96     97* 98* 99* 100* 101* 102* 103* 104* 105* 106*
      107* 108* 109     110* 111* 112* 113* 114     115* 116* 117* 118*
      119* 120* 121* 122* 123* 124* 125* 126* 127* 128* 129     130*
      131* 132* 133* 134* 135* 136* 137* 138     139* 140* 141* 142*
      143* 144
G1>>

```

This is a chart of all of the groups in the system. The first column gives the order of the group. The numbers of the groups of that order are found to the right. Notice that there is only one group of orders 2, 3, 5, 7, 11. Does this suggest any conjecture?

Here we find

```
G1>> CHART Order of Groups (1-32 or 0)
Number 12
20 21 22* 23* 24*
There are 5 Groups of order 12
2 abelian and 3 non-abelian
G1>> CHART Order of Groups (1-32 or 0)
Number 6
7 8*
There are 2 Groups of order 6
      1 abelian and 1 non-abelian
```

We can investigate the groups of order 6 by looking at the orders of the elements:

```
G1>> ORDERS    for Group Number 7

Group number 7 of Order 6
  1 elements of order  1:  A
  1 elements of order  2:  D
  2 elements of order  3:  C E
  2 elements of order  6:  B F
```

```
G7>> ORDERS    for Group Number 8

Group number 8 of Order 6
  1 elements of order  1:  A
  3 elements of order  2:  D E F
  2 elements of order  3:  B C
  0 elements of order  6:
```

G8>>

Notice that group 7 has elements of order 6. This means that group 7 is cyclic. It is generated by B or by F. This can also be seen by looking at the powers of B (or powers of F) to see that they give the entire group. Notice that the prompt shows that the current group is group 8, so we switch to group 7 by using the GROUP command.

```
G8>> GROUP    Group Number 7
G7>> POWERS   for element B
A B C D E F
G7>> POWERS   for element F
A F E D C B
```

This means that group 7 is isomorphic to  $Z_6$ .

Group 8 must be  $S_3$ . To see this we go to the PERMGROUPS submenu. Notice that  $S_3$  is generated by  $(1\ 2\ 3)$  and  $(1\ 2)$ .

G7>> PERMGRPS

CREATE	ELEMENTS	HELP	INFO
INSTALL	MAIN	MULTIPLY	QUIT
STOP	X		

PERM>> X

This will provide information about the next command you use. INFO and X do the same thing but X is quicker to type.

PERM>> CREATE

This will determine the subgroup of  $S_n$  generated by a given set of permutations (given as a product of cycles). You must put in  $n$  (for  $S_n$ ) and then the generators using numbers  $1..n$  for example  $(1\ 2)(3\ 4\ 5)$ . The program will only compute groups up to order 51. If the resulting group has order 32 or less, you can install the table as one of the groups 1-5.

We get a description of the CREATE command by typing “X” first and then CREATE. (You can also use the command INFO first but “X” is quicker to type.)

PERM>> CREATE

Subgroup of  $S_n$  -- what is  $n$ ? Number 3

Put in generators as product of cycles.  
End with a blank line

Generator (1 2 3)

Generator (1 2)

Generator

Group is of order 6

A ( )	B (2 3 )	C (1 2 )
D (1 2 3 )	E (1 3 2 )	F (1 3 )

Notice that we ended the input of generators by pressing Enter on a blank line.

PERM>> INSTALL

Install as table  $k$  (1..5) Number 1

After creating this group (using permutations) we INSTALL it as group 1. This replaces group 1 (which was the trivial group) by the group we just created. This lets us return to the MAIN menu and apply the main commands.

PERM>> MAIN

CENTER	CENTRALIZER	CHART	CONJ-CLS
COSETS	EVALUATE	EXAMPLES	GENERATE
GROUP	HELP	IDENTIFY	INFO
ISOMORPHISM	LEFT	NORMALIZER	NUMTHEORY
ORDERS	PERMGRPS	POWERS	QUIT
RIGHT	SEARCH	STOP	SUBGROUPS
TABLE	X		

```
G1>> IDENTIFY    Group Number 1
      G32 number:8    Order:6
      GAP SmallGroup(6,1)
      Generators:
B = (1 2 )(3 4 )(5 6 )
C = (1 3 )(2 5 )(4 6 )
```

The IDENTIFY command is particularly useful for determining which numbered group corresponds to a group created by permutations. Here we see that the group we created is, in fact, group 8. We also find that generators are B and C – which are of order 2. The generators given are not the smallest possible. They come from treating the generators as permutations of the group elements by left multiplication. They are elements of  $S_6$  since the group has 6 elements. This suggests, however, that our group is also generated by two elements of order 2. We can verify this by returning to PERMGRPS.

```
G1>> PERMGRPS
```

CREATE	ELEMENTS	HELP	INFO
INSTALL	MAIN	MULTIPLY	QUIT
STOP	X		

```
PERM>> CREATE
```

```
Subgroup of Sn -- what is n? Number 3
Put in generators as product of cycles.
End with a blank line
```

```
Generator (1 2)
```

```
Generator (1 3)
```

```
Generator
```

```
Group is of order 6
```

A ( )	B (2 3 )	C (1 2 )
D (1 2 3 )	E (1 3 2 )	F (1 3 )

```
PERM>> INSTALL
```

```
Install as table k (1..5) Number 1
```

```
PERM>> MAIN
```

CENTER	CENTRALIZER	CHART	CONJ-CLS
COSETS	EVALUATE	EXAMPLES	GENERATE
GROUP	HELP	IDENTIFY	INFO
ISOMORPHISM	LEFT	NORMALIZER	NUMTHEORY
ORDERS	PERMGRPS	POWERS	QUIT
RIGHT	SEARCH	STOP	SUBGROUPS
TABLE	X		

```
G1>> IDENTIFY    Group Number 1
```

```
G32 number:8    Order:6
```

```
GAP SmallGroup(6,1)
```

```
Generators:
```

```
B = (1 2 )(3 4 )(5 6 )
```

```
C = (1 3 )(2 5 )(4 6 )
```

We can find an explicit isomorphism between group 1 and group 8 by using the ISOMORPHISM command. This allows you to map elements of the first group to the second – and it computes the consequence of your assignment.

```
G1>> ISOMORPHISM    from Group Number 1 to Group Number 8
```

You will find that you cannot send an element of group 1 to an arbitrary element of group 8. I will not show all the steps, but here is one possible isomorphism. The elements of the top row are sent to the corresponding elements of the second row.

```
A B C D E F
A D F C B E
*** SUCCESS ***
```

It might help to know that an isomorphism must send any element of the first group to an element of the same order in the second group.

```
G1>> ORDERS    for Group Number 1
```

```
Group number 1 of Order 6
  1 elements of order  1:  A
  3 elements of order  2:  B C F
  2 elements of order  3:  D E
  0 elements of order  6:
```

```
G1>> ORDERS    for Group Number 8
```

```
Group number 8 of Order 6
  1 elements of order  1:  A
  3 elements of order  2:  D E F
  2 elements of order  3:  B C
  0 elements of order  6:
```

Thus we can send B in group 1 possibly to D, E, or F in group 8. Notice that the isomorphism we found sends B to D. An isomorphism must also send generators of the first group to generators of the second group. We can use the GENERATE command to see the B and C generate group 1:

```
G8>> GROUP    Group Number 1
G1>> GENERATE    a subgroup using set: { bc }
{ A B C D E F }
```

Once we know where B and C go, the mapping of the other elements follows. We can also find generators using the SUBGROUPS command.

```
G1>> SUBGROUPS    of Group Number 1
... wait
```

```

* = Normal subgroup
Generators      Subgroup
0 { }           *{ A }
1 { B }         { A B }
2 { C }         { A C }
3 { F }         { A F }
4 { D }         *{ A D E }
5 { B C }       *{ A B C D E F }
```

We have not mentioned  $Z_3 \times Z_2$ . This should be an abelian group of order 6, so it must be  $Z_6$ . We have Theorem: if  $G$  is abelian,  $o(a)=m$ ,  $o(b)=n$  and  $\gcd(m,n)=1$  then  $o(ab)=mn$ . We see that in group 7 (which is  $Z_6$ ) the product of an element of order 2 and an element of order 3 is an element of order 6.

```
G1>> ORDERS    for Group Number 7
```

```
Group number 7 of Order 6
  1 elements of order  1:   A
  1 elements of order  2:   D
  2 elements of order  3:  C E
  2 elements of order  6:  B F
```

```
G7>> EVALUATE    (use ' for inverse) dc= F
```

```
G7>> EVALUATE    (use ' for inverse) de= B
```

(Note that Groups32 is not case sensitive – so  $dc$  is the same as  $DC$ .) We contrast this with group 8 (which is  $S_3$ ).

```
G7>> ORDERS    for Group Number 8
```

```
Group number 8 of Order 6
  1 elements of order  1:   A
  3 elements of order  2:  D E F
  2 elements of order  3:  B C
  0 elements of order  6:
```

```
G8>> EVALUATE    (use ' for inverse) db= E
```

In this group,  $o(D)=2$ ,  $o(B)=3$  but  $o(DB)=2$ . There is no element of order 6!

While we are classifying the first few groups, notice that there are two groups of order 4 – both abelian.

```
G1>> CHART      Order of Groups (1-32 or 0) Number 4
      4      5
      There are 2 Groups of order 4
      2 abelian and 0 non-abelian
```

```
G1>> ORDERS    for Group Number 4
```

```
Group number 4 of Order 4
  1 elements of order  1:   A
  1 elements of order  2:   C
  2 elements of order  4:  B D
```

```
G4>> ORDERS    for Group Number 5
```

```
Group number 5 of Order 4
  1 elements of order  1:   A
  3 elements of order  2:  B C D
  0 elements of order  4:
```

Group 4 has elements of order 4, so it is cyclic (it is  $Z_4$ ). Group 5 has no elements of order 4. It is the other group of order 4, namely  $Z_2 \times Z_2$ .



There is only one group of order  $n$  for  $n = 2, 3, 5, 7$ . This group is cyclic and must be  $Z_n$ .

THUS WE NOW KNOW ALL THE GROUPS NUMBERED 1..9 OF ORDERS 1..7.

We now look at groups of order 8.

```
G8>> CHART ORDER OF GROUPS (1-32 OR 0) NUMBER 8
      10  11  12  13* 14*
      THERE ARE 5 GROUPS OF ORDER 8
      3 ABELIAN AND 2 NON-ABELIAN
```

By the Fundamental Theorem of Abelian Groups we know that any finite abelian group is the direct product of cyclic groups (of prime power order). The abelian groups of order 8 must be  $Z_8$  or  $Z_4 \times Z_2$  or  $Z_2 \times Z_2 \times Z_2$ . We look at the orders of the elements for groups 10, 11 and 12.

```
G8>> ORDERS for Group Number 10
```

```
Group number 10 of Order 8
  1 elements of order 1:  A
  1 elements of order 2:  E
  2 elements of order 4:  C G
  4 elements of order 8:  B D F H
```

```
G10>> ORDERS for Group Number 11
```

```
Group number 11 of Order 8
  1 elements of order 1:  A
  3 elements of order 2:  C E G
  4 elements of order 4:  B D F H
  0 elements of order 8:
```

```
G11>> ORDERS for Group Number 12
```

```
Group number 12 of Order 8
  1 elements of order 1:  A
  7 elements of order 2:  B C D E F G H
  0 elements of order 4:
  0 elements of order 8:
```

Group 10 has elements of order 8, so this must be  $Z_8$ . Group 11 has no element of order 8, but it has elements of order 4. So group 11 is  $Z_4 \times Z_2$ . Finally, group 12 has no elements of order 8 or 4 but many elements of order 2. So group 12 is  $Z_2 \times Z_2 \times Z_2$ .

Notice that we have been able to determine the decomposition of a finite abelian group of order 8 by looking at the distribution of the orders of its elements. We may ask whether this is true in general. If we know the number of elements of each order in an abelian group  $G$ , can we determine how it can be represented as a product of cyclic groups. Actually it is also interesting to ask, conversely, if we can determine the distribution of orders of elements from the representation of  $G$  as the direct product of cyclic groups.

At any rate, we have just identified groups 10, 11 and 12 (the abelian groups of order 8). An interesting collection of groups is the dihedral groups,  $D_n$ .  $D_n$  is the group of symmetries of a regular  $n$ -gon. This group is generated by a rotation and a "flip". If the vertices of the  $n$ -gon are numbered 1— $n$ , let's say clockwise, then the clockwise rotation is given by the permutation  $(1\ 2\ \dots\ n)$  which sends 1 to 2, 2 to 3, etc. We have several choices for a flip. If we keep vertex 1 fixed and flip in the axes going through 1, the flip is given by the permutation  $(2\ n)(3\ n-1)\dots$ .  $D_n$  is a group of order  $2n$ . We examine  $D_4$  which is a non-abelian group of order 8. This is generated by the permutations  $(1\ 2\ 3\ 4)$  and  $(2\ 4)$ . Go through PERMGRPS to generate this group and install it as group 1.

G12>> PERMGRPS

CREATE	ELEMENTS	HELP	INFO
INSTALL	MAIN	MULTIPLY	QUIT
STOP	X		

PERM>> CREATE

Subgroup of  $S_n$  -- what is n? Number 4  
Put in generators as product of cycles.  
End with a blank line

Generator (1 2 3 4)

Generator (2 4)

Generator

Group is of order 8

A ( )	B (2 4 )	C (1 2 )(3 4 )
D (1 2 3 4 )	E (1 3 )	F (1 3 )(2 4 )
G (1 4 3 2 )	H (1 4 )(2 3 )	

PERM>> INSTALL

Install as table k (1..5) Number 1

PERM>> MAIN

CENTER	CENTRALIZER	CHART	CONJ-CLS
COSETS	EVALUATE	EXAMPLES	GENERATE
GROUP	HELP	IDENTIFY	INFO
ISOMORPHISM	LEFT	NORMALIZER	NUMTHEORY
ORDERS	PERMGRPS	POWERS	QUIT
RIGHT	SEARCH	STOP	SUBGROUPS
TABLE	X		

G1>> IDENTIFY Group Number 1

G32 number:13 Order:8

GAP SmallGroup(8,3)

Generators:

B = (1 2 )(3 4 )(5 6 )(7 8 )

C = (1 3 )(2 7 )(4 5 )(6 8 )

G1>> ORDERS for Group Number 1

Group number 1 of Order 8

1 elements of order 1: A  
5 elements of order 2: B C E F H  
2 elements of order 4: D G  
0 elements of order 8:

A ( )	B (2 4 )	C (1 2 )(3 4 )
D (1 2 3 4 )	E (1 3 )	F (1 3 )(2 4 )
G (1 4 3 2 )	H (1 4 )(2 3 )	

Notice that D and G are rotations. B,C,E,F,H are flips. It might help to draw a square on a piece of paper and see geometrically what each of these permutations is. One thing to observe is that if we flip, then rotate, then flip back we get the rotation in the other direction.

G1>> EVALUATE (use ' for inverse) bdb= G

The group  $D_n$  is generated by two elements  $r, f$ .  $r$  is of order  $n$ ,  $f$  is of order 2. We have a relation between the generators:  $frf=r'$  (where  $'$  indicates inverse). The SEARCH command will find groups which have given generators and relations. Here is how to get a list of all dihedral groups. Notice that we specify that  $f$  is of order 2, but we do not specify the order of  $r$  – so we get all dihedral groups.

```
G1>> SEARCH
Enter distinct generators as a string
e.g. RS means two generators R and S
  Generators: fr
Do you want these to generate the entire group?(y or n) Y

Enter the exact order for each generator.
Press Enter for no order specified
  F is of order 2
  R is of order

A relation is of the form LHS = RHS
Put in LHS RHS or LHS ( if RHS is e )
  <Press ENTER to quit>

LHS RHS >> frf r'
LHS RHS >>
Generators:
  FR
Orders:
  F= 2

RELATIONS:
  FRF' = R'
```

-- Pressing ESC will abort the search --

```
1 group order = 8 F = B R = D
2 group order = 2 F = B R = B
5 group order = 4 F = C R = B
8 group order = 6 F = D R = B
13 group order = 8 F = E R = B
18 group order = 10 F = F R = B
22 group order = 12 F = G R = B
27 group order = 14 F = H R = B
40 group order = 16 F = I R = B
47 group order = 18 F = J R = B
52 group order = 20 F = K R = B
58 group order = 22 F = L R = B
69 group order = 24 F = M R = B
78 group order = 26 F = N R = B
86 group order = 28 F = O R = B
92 group order = 30 F = P R = B
111 group order = 32 F = B R = D
144
0.202 seconds
```

We see that group 13 is on the list as  $D_4$ . (It also appears as group 1 because we installed it as group 1 when we studied its permutation representation.)

Here are the orders and subgroups of group 14:

```
G1>> ORDERS    for Group Number 14
```

```
Group number 14 of Order 8
  1 elements of order  1:  A
  1 elements of order  2:  C
  6 elements of order  4:  B D E F G H
  0 elements of order  8:
```

```
G14>> SUBGROUPS  of Group Number 14
... wait
```

```
  * = Normal subgroup
Generators      Subgroup
0 { }           *{ A }
1 { C }         *{ A C }
2 { B }         *{ A B C D }
3 { E }         *{ A C E G }
4 { F }         *{ A C F H }
5 { B E }       *{ A B C D E F G H }
```

This is the first example of a non-abelian group all of whose subgroups are normal (such groups are called Hamiltonian). This group is generated by two elements. One set of generators is b and e. Since the subgroup generated by b is normal,  $ebe'$  must be in that subgroup. Since b is of order 4,  $ebe'$  must be of order 4. If  $ebe' = b$  then  $eb = be$  so e would commute with b. We use the CENTRALIZER command to show that this is not true. So  $ebe' = d = b'$ . We actually can show this using the EVALUATE command.

```
G14>> CENTRALIZER  of element B in current group
{ A B C D }
G14>> EVALUATE    (use ' for inverse) ebe' = D
G14>> EVALUATE    (use ' for inverse) b' = D
```

We have found a relation,  $ebe' = b'$  between the generators. Use this in the SEARCH command.

```
G14>> SEARCH
Enter distinct generators as a string
e.g. RS means two generators R and S
  Generators: be
Do you want these to generate the entire group?(y or n) Y

Enter the exact order for each generator.
Press Enter for no order specified
  B is of order 4
  E is of order 4

A relation is of the form LHS = RHS
Put in LHS RHS or LHS ( if RHS is e )
  <Press ENTER to quit>

LHS RHS >> ebe' b'
LHS RHS >>
```

Generators:

BE

Orders:

B= 4

E= 4

RELATIONS:

EBE' = B'

-- Pressing ESC will abort the search --

14 group order = 8 B = E E = B

38 group order = 16 B = J E = B

144

0.121 seconds

Group 14 does satisfy the generator/relations but another group does as well. However the same reasoning as above shows that  $beb' = e'$ . We use these two relations in a SEARCH.

G14>> SEARCH

Enter distinct generators as a string

e.g. RS means two generators R and S

Generators: be

Do you want these to generate the entire group?(y or n) Y

Enter the exact order for each generator.

Press Enter for no order specified

B is of order 4

E is of order 4

A relation is of the form LHS = RHS

Put in LHS RHS or LHS ( if RHS is e )

<Press ENTER to quit>

LHS RHS >> ebe' b'

LHS RHS >> beb' e'

LHS RHS >>

Generators:

BE

Orders:

B= 4

E= 4

RELATIONS:

EBE' = B'

BEB' = E'

-- Pressing ESC will abort the search --

14 group order = 8 B = E E = B

144

0.120 seconds

Group 14 has another presentation. The set  $\{\pm 1, \pm i, \pm j, \pm k\}$  can be equipped with a multiplication in which  $i^2 = j^2 = k^2 = -1$ ,  $ij = k$ ,  $ji = -k$ ,  $jk = i$ ,  $kj = -i$ ,  $ki = j$ ,  $ik = -j$ . This looks like the familiar cross product except the squares are -1 rather than 0. [The quaternions are a number system whose elements are  $a + bi + cj + dk$  with  $a, b, c, d$  real. The multiplication of quaternions is determined by the given relations. The invertible elements (units) in the quaternions are  $\pm 1, \pm i, \pm j, \pm k$  so group 14 is the group of quaternionic units.] We can use the SEARCH command to see that group 14 is generated by  $i, j, k$  of order 4 with relations  $ij=k$ ,  $jk=i$ ,  $ki=j$ .

```
G1>> SEARCH
Enter distinct generators as a string
e.g. RS means two generators R and S
    Generators:  ijk
Do you want these to generate the entire group?(y or n) Y

Enter the exact order for each generator.
Press Enter for no order specified
    I is of order      4
    J is of order      4
    K is of order      4

A relation is of the form  LHS = RHS
Put in LHS RHS  or  LHS  ( if RHS is e )
    <Press ENTER to quit>

LHS RHS >>  ij k
LHS RHS >>  jk i
LHS RHS >>  ki j
LHS RHS >>
Generators:
    IJK
Orders:
    I= 4
    J= 4
    K= 4

RELATIONS:
    IJ    = K
    JK    = I
    KI    = J

-- Pressing ESC will abort the search --

    14  group order =  8  I = F    J = E    K = B
144
10.022 seconds
```

We have now identified the groups of order 8.

We now say a few things about the COSETS command. Let us go back to group 8 (which is  $S_3$ ).

```
G8>> SUBGROUPS    of Group Number 8
... wait
```

	Generators	Subgroup
0	{ }	*{ A }
1	{ D }	{ A D }
2	{ E }	{ A E }
3	{ F }	{ A F }
4	{ B }	*{ A B C }
5	{ B D }	*{ A B C D E F }

Notice that subgroups 1,2 and 3 are not normal subgroups while subgroup 4 is normal. One characterization of a normal subgroup is that every right coset is a left coset. We see, using subgroup 1 (generated by D) that the left and right cosets are not the same.

```
G8>> COSETS      of subg generated by set: { d }
```

Left Cosets	Right Cosets
{ A D }	{ A D }
{ B F }	{ B E }
{ C E }	{ C F }

The subgroup { A D } is NOT a NORMAL subgroup

We can try to make the cosets of a subgroup, H, into a group by setting  $Hx * Hy = Hxy$ . The problem is that  $Hx$  is also the right coset  $Hx$  for any  $x$  in the coset. The multiplication has us take any representative of the first coset and multiply by a representative of the second coset. This gives a well-defined multiplication if the product always lands in the same coset. This happens when H is a normal subgroup. We see that this is not the case in this example.

```
G8>> EVALUATE    (use ' for inverse) bb= C
G8>> EVALUATE    (use ' for inverse) ee= A
```

b and e both represent the same right coset. We get different cosets if we square b than we get if we square e. Thus the multiplication of cosets is not well-defined. Check what happens if you use cosets for subgroup 4 (which is normal). This is the subgroup generated by b – which you use to identify the subgroup to the COSETS command.

The NORMALIZER of a subgroup H is the largest subgroup in which H is normal.

```
G13>> SUBGROUPS    of Group Number 13
... wait
```

```

* = Normal subgroup
Generators      Subgroup
0 { }           *{ A }
1 { C }         *{ A C }
2 { E }         { A E }
3 { F }         { A F }
4 { G }         { A G }
5 { H }         { A H }
6 { B }         *{ A B C D }
7 { C E }       *{ A C E G }
8 { C F }       *{ A C F H }
9 { B E }       *{ A B C D E F G H }
```

```
G13>> NORMALIZER    of subgroup gen by set: { h }
{ A C F H }
```

Notice that subgroup 5 is a normal subgroup of subgroup 8 (which is its normalizer). Subgroup 8 is a normal subgroup of G. This is an example of subgroups K and H of a group G where K is normal in H, H is normal in G, but K is not normal in G. This settles a question asked in an algebra text.

When you use Groups32, make use of the fact that “H” gives the menu of commands and “X” or “INFO” followed by a command gives information about that command. NUMTHEORY provides a submenu of commands dealing with the integers:

Integers must be between -2147483647 and 2147483647

ADD-TABLE	CALCULATOR	CHINESE	FACTOR
GCD	HELP	INFO	INVERSE
MAIN	MODULUS	MULT-TABLE	N^K
POWERS	QUIT	SIEVE	STATUS
STOP	UNITS	VGCD	X

You can use “X” or “INFO” to explore these commands.