Practice Questions

1. Find the limits.
   
   a) \( \lim_{x \to \infty} x \cdot \sin \frac{1}{x} \) and \( \lim_{x \to 0} x \cdot \sin \frac{1}{x} \)
   
   b) \( \lim_{x \to 0} x^{\sin x} \)
   
   c) \( \lim_{x \to 0} \frac{\tan x}{x} \)
   
   d) \( \lim_{x \to 16} \frac{\sqrt{x - 4}}{x - 16} \)
2. Show $e^x = -\ln x$ has a solution in $(0, 1)$. (Hint: IVT).

3. Find $g'(b)$ and $g''(b)$ where $g$ is the inverse of $f$.

   a) $f(x) = 4x^3 - 2x$ \quad b = -2.

   b) $f(x) = e^x$ \quad b = e.
4. Katie and Franklin are in kayaks in the middle of Mission Bay. At time \( t = 0 \), Katie paddle south at a speed of 5 mph and Franklin paddles towards the east at 3 mph.

a) How far have Katie & Franklin traveled after 12 minutes?

b) At what rate is the distance between them increasing at \( t = 12 \text{ min} \).
5. Evaluate the following derivatives:

a) \( \frac{d}{dx} \int_0^{x^2} e^{4t^2} \, dt \)

b) \( \frac{d}{dx} \sqrt{\sin(3x+1)} \cdot e^x \)

c) \( \frac{d}{dx} \frac{\sin(2x)}{e^x - 1} \)
6. Show that the equation \(4x^5 + x^3 + 2x + 1 = 0\) has only one root in \((-\infty, \infty)\). In other words, show that it cannot have two roots and has at least one.
7. Evaluate the following integrals

a) \[ \int_0^7 \sqrt{49 - x^2} \, dx \]

b) \[ \int_{-4}^3 1 \times 1 \, dx \]

c) \[ \int_0^1 (e^{2x} + 1) \, dx \]