Math 20C  Summer Session II  2010  Prof. Shen

Exam 2  Review (Supplement & Ch. 14)

Supplement

Let \( r(t) = \langle x(t), y(t) \rangle \). Then unit tangent = \( \frac{\langle x'(t), y'(t) \rangle}{\sqrt{x'(t)^2 + y'(t)^2}} \).

If unit tangent = \( \langle a, b \rangle \) then unit normal = \( \langle -b, a \rangle \).

Curvature

\[
K(x) = \frac{x''(t)y'(t) - y''(t)x'(t)}{(x'(t)^2 + y'(t)^2)^{3/2}}
\]

Positive curvature  \( \frac{d}{dr} \)  \( r \)

Curves of curvature

Radius \( \rho = \frac{1}{K} \) \( \rho \) curvature \( \pm \) depending on \( t \)

Acceleration \( \vec{a} = \frac{d^2 \vec{r}}{dt^2} = \vec{v} \times \vec{a} + \vec{a} \times \vec{v} + \frac{d^2}{dt^2} \vec{r} \)

Curvilinear acceleration

\[
\text{acceleration} = S'' T + K(S')^2 N
\]

14.1 Level curves = contour diagram = horizontal cross section

Let \( z = c \) (a constant) plot in \( xy \) plane.

Get vertical cross sections by letting \( x, y = \) constant.

14.2 Limits → if continuous pug in point can split up sums / products / constants as long as limits all exist.

14.3 Partial derivatives \( \frac{\partial f}{\partial x} \) → treat everything besides \( x \) as a constant → tells you the rate of change of \( f \) when you move in \( x \) direction.

Higher order partials \( f_{xy} = f_{yx} \)

\( f_{xy} \) means first take derivative \( \frac{df}{dx} \), let \( \frac{df}{dx} = g \) then do \( \frac{dg}{dy} \)

14.4 Linearization → use tangent plane to approximate function.

\[
L(x, y) = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b)
\]

Tangent plane \( z = \) “

Prox Error = \( f_x(a, b)(x-a) + f_y(a, b)(y-b) \)
14.5 Gradient

\[ \nabla f = <f_x(a,b), f_y(a,b)> \]

- Points in direction of max rate of increase
- \(-\nabla f\) points in direction of max rate of decrease

\[
\|\nabla f\| = \text{max rate}
\]

**Directional Derivative** (rate of change in direction of \(\vec{V}\))

\[
D_{\vec{V}}f(a,b) = \frac{\nabla f(a,b) \cdot \vec{V}}{\|\vec{V}\|}
\]

4.7

Global min/max occur when \(f_x = f_y = 0\) or when one of \(f_x\) or \(f_y\) DNE

2nd derivative test

\[
\begin{vmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{vmatrix} > 0 \quad f_{xx} > 0 \quad \text{min}
\]

\[
\begin{vmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{vmatrix} < 0 \quad f_{xx} < 0 \quad \text{max}
\]

\[
\begin{vmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{vmatrix} = 0 \quad \text{saddle}
\]

\[
\begin{vmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{vmatrix} = 0 \quad \text{inconclusive}
\]

Global Extrema - at CPs or on boundary.

4.8 Lagrange Multipliers

Find min/max of \(f\) w/ constraint \(g\).

- CPs occur when \(\nabla f = \lambda \nabla g\)
  
  Write out eqn

1) Solve for \(\lambda\), set equal get \(f\) & \(g\) in terms of each other.

2) Plug into \(g\).

3) Solve \(x, y, z\) to determine if min or max.