A linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is completely determined by its effect on columns of the $n \times n$ identity matrix. TRUE The columns on the identity matrix are the basis vectors in $\mathbb{R}^n$. Since every vector can be written as a linear combination of these, and $T$ is a linear transformation, if we know where these columns go, we know everything.

If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ rotates vectors about the origin through an angle $\phi$, then $T$ is a linear transformation. TRUE. To show this we would show the properties of linear transformations are preserved under rotations.

When two linear transformations are performed one after another, then combined effect may not always be a linear transformation. FALSE Again, check properties to show it is a linear transformation.
A mapping \( T : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is onto \( \mathbb{R}^m \) if every vector \( \mathbf{x} \) in \( \mathbb{R}^n \) maps onto some vector in \( \mathbb{R}^m \). FALSE A linear transformation is onto if the codomain is equal to the range.

If \( A \) is a \( 3 \times 2 \) matrix, the transformation \( \mathbf{x} \rightarrow A\mathbf{x} \) cannot be one-to-one. FALSE Since the transformation maps from \( \mathbb{R}^2 \) to \( \mathbb{R}^3 \) and \( 2 < 3 \) it can be one-to-one but not onto.
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- Not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation. FALSE For a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ we see where the basis vector in $\mathbb{R}^n$ get mapped to. These form the standard matrix.

- The columns of the standard matrix for a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images of the columns of the $n \times n$ identity matrix. TRUE

- The standard matrix of a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that reflects points through the horizontal axis, the vertical axis, or the origian has the form $\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$, where $a$ and $d$ are $\pm 1$ TRUE We can check this by checking the images of the basis vectors.
A mapping \( T : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is one-to-one if each vector in \( \mathbb{R}^n \) maps onto a unique vector in \( \mathbb{R}^m \). FALSE A mapping is one-to-one if each vector in \( \mathbb{R}^m \) is mapped to from a unique vector in \( \mathbb{R}^n \).

If \( A \) is a \( 3 \times 2 \) matrix, then the transformation \( \mathbf{x} \rightarrow A\mathbf{x} \) cannot map \( \mathbb{R}^2 \) onto \( \mathbb{R}^3 \). TRUE You cannot map a space of lower dimension onto a space of higher dimension.
If \( A \) and \( B \) are \( 2 \times 2 \) with columns \( \mathbf{a}_1, \mathbf{a}_2 \) and \( \mathbf{b}_1, \mathbf{b}_2 \) then \( AB = [\mathbf{a}_1 \mathbf{b}_1, \mathbf{a}_2 \mathbf{b}_2] \). FALSE Matrix multiplication is ”row by column”.

Each column of \( AB \) is a linear combination of the columns of \( A \) using weights from the corresponding column of \( B \). FALSE Swap \( A \) and \( B \) then its true

\[ AB + AC = A(B + C) \] TRUE Matrix multiplication distributes over addition.

\[ A^T + B^T = (A + B)^T \] TRUE See properties of transposition. Also should be able to think through to show this. When we add we add corresponding entries, these will remain corresponding entries after transposition.

The transpose of a product of matrices equals the product of their tranposes in the same order. FALSE The transpose of a product of matrices equals the product of their tranposes in the reverse order.
If $A$ and $B$ are $3 \times 3$ and $B = [b_1 \ b_2 \ b_3]$, then $AB = [Ab_1 + Ab_2 + Ab_3]$. FALSE This is right but there should not be +'s in the solution. Remember the answer should also be $3 \times 3$.

The second row of $AB$ is the second row of $A$ multiplied on the right by $B$. TRUE

$(AB)C = (AC)B$ FALSE Matrix multiplication is not commutative.

$(AB)^T = A^T \ast B^T$ FALSE $(AB)^T = B^T \ast A^T$

The transpose of a sum of matrices equals the sum of their transposes. TRUE