Midterm 1 Practice

In studying for the exam, I encourage you to:

- Understand how to do all of the homework problems given so far.
- Understand the problems from the second worksheet, particularly problems 1 through 4.
- Know definitions.
- Do the following problems! Treat this as a practice exam. We will go over the answers to this in class on Monday.

1. (a) Give the definition of an isomorphism of binary structures.
   (b) Is $\phi : \mathrm{GL}_2(\mathbb{R}) \to \mathbb{R}^\times$ given by $\phi(A) = \det A$ an isomorphism of the binary structures $(\mathrm{GL}_2(\mathbb{R}), \times)$ and $(\mathbb{R}^\times, \times)$?
2. (a) Let G be a group. Give the definition of a subgroup of G.
   (b) Is $S = \{A \in \mathrm{GL}_2(\mathbb{R}) \mid \det A = 2\}$ a subgroup of $\mathrm{GL}_2(\mathbb{R})$?
   (c) Is $(S = \{2^n \mid n \in \mathbb{Z}\}$ a subgroup of $(\mathbb{Q}^\times, \times)$?
   (d) Find all subgroups of the group $\mathbb{Z}_5$ and give all generators for each subgroup.
   (e) Find all subgroups of the group $\mathbb{Z}_9$ and give all generators for each subgroup.
3. (a) Give the definition of an abelian group.
   (b) If G is an abelian group with identity e, show that $H = \{x \in G \mid x^2 = e\}$ is a subgroup of G.
4. (a) Give the definition of a cyclic group.
   (b) If G is a cyclic group and $\phi : G \to G'$ is an isomorphism of G with another group G', prove that G' is also cyclic.