Your Name

PID

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- Check that you have a complete exam. There are 4 questions for a total of 26 points.
- Each question should take up roughly 12 minutes of your time. Do not spend too much time on each question.
- You are allowed to have one single sided, handwritten note sheet.
- Cheating will result in a zero and be reported to the University.
- **Show all of your work**. You should prove your answers: for example, if a problem asks "Is .... a group?" a yes or no answer is not enough; you must justify why it is true. If you use a main result from class, be sure to clearly state the result.
- If you need more space to answer a question, continue on the back of the page, and indicate that you have done so.
- GOOD LUCK!

Question	Points	Score
1	5	
2	6	
3	7	
4	8	
Total:	26	

1. (a) (1 point) Let A be a set. Give the definition of a permutation of A.

   (b) (2 points) Let $A = \mathbb{Z}$. Is the function $\phi : A \to A$ given by $\phi(n) = 5n$ a permutation of A?

   (c) (2 points) Let $A = \mathbb{Z}_6$. Is the function $\phi : A \to A$ given by $\phi(n) = 5n \mod 6$ a permutation of A?

2. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 6 & 5 & 4 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix}$ be permutations in S_6.

   (a) (2 points) Write σ and τ in cycle notation.

   (b) (2 points) Determine if σ and/or τ is an element of A_6.

   (c) (2 points) Compute $\sigma\tau$ and find the order of $\sigma\tau$.

3. (a) (1 point) If H is a subgroup of a group G, give the definition of a left coset of H.

   (b) (3 points) Let $M_n = \{a \in \mathbb{Z}_n \mid \gcd(a, n) = 1\}$. In class, we showed that $(M_n, \cdot_n)$ is a group, where $\cdot_n$ is multiplication modulo n. Prove that

$$H = \{a \in M_n \mid a \cdot_n a = 1\}$$

   is a subgroup of M_n.

   (c) (3 points) Determine all left cosets of H in M_7, where H is the subgroup from part (b).

4. (a) (1 point) State Lagrange's Theorem.

   (b) (3 points) Let G be a group of order pq, where p and q are prime numbers. Prove that every proper subgroup of G is cyclic.

   (c) (4 points) If G is a finite abelian group with two elements x, y of order 2 and $x \neq y$, use Lagrange's Theorem to prove that $|G|$ is a multiple of 4. (Hint: find a subgroup of order 4.)