# MIDTERM 2 PRACTICE

In studying for the exam, I encourage you to:

- Understand how to do all of the homework problems from Homework 3 and Homework 4.
- Understand the problems from the worksheet on cosets.
- Know definitions.
- Do the following problems! Treat this as a practice exam. We will go over the answers to this in class on Wednesday.

1. (a) Let $A$ be a set. Give the definition of a permutation of $A$.
   (b) Let $A = \mathbb{Z}$. Which of the following are permutations of $A$?
      i. The function $\sigma : A \to A$ given by $\sigma(n) = 3n - 2$
      ii. The function $\tau : A \to A$ given by $\tau(n) = |n|$

2. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ be permutations in $S_6$.
   (a) Write $\sigma$ and $\tau$ in cycle notation.
   (b) Determine if $\sigma$ and $\tau$ are elements of $A_6$.
   (c) Find $\sigma\tau$ and $|\langle \sigma\tau \rangle|$.

3. (a) Show that the set $H = \{\sigma \in S_5 \mid \sigma(5) = 5\}$ is a subgroup of $S_5$.
   (b) Show that $S_4 \cong H$.
   (c) If $H$ is a subgroup of a group $G$, give the definition of a left coset of $H$.
   (d) Show that $H' = \{\sigma \in S_5 \mid \sigma(5) = 1\}$ is a left coset of $H$.

4. (a) State Lagrange's Theorem.
   (b) If $G$ is a group with $p$ elements and $p$ is a prime number, prove that $G$ is abelian.
   (c) Let $M_n = \{a \in \mathbb{Z}_n \mid \gcd(a, n) = 1\}$. In class we showed that $(M_n, \cdot_n)$ is a group. Prove that, for $n \geq 3$, $|M_n|$ is even.