Homework 3: Due Thursday, February 13

1. **Problem 1.** Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$
   (a) Section 8, Problem 1: Compute $\tau\sigma$.
   (b) Section 8, Problem 2: Compute $\tau^2\sigma$.
   (c) Section 8, Problem 4: Compute $\sigma^{-2}\tau$.
2. **Problem 2.** For σ and τ as in Problem 1, compute the following.
   (a) Section 8, Problem 6: $|\langle\sigma\rangle|$.
   (b) Section 8, Problem 7: $|\langle\tau^2\rangle|$.
   (c) Section 8, Problem 8: σ^{100}.
3. **Problem 3.**
   (a) Section 8, Problem 16: Find the number of elements in the set $\{\sigma \in S_4 \mid \sigma(3) = 3\}$.
   (b) Show that the set from part (a) is a subgroup of S_4.
4. **Problem 4.** Section 9, Problem 7. Consider the permutation $\sigma = (145)(78)(257) \in S_8$. Write σ in cycle notation.
5. **Problem 5.**
   (a) Section 9, Problem 14: Find the maximum possible order for an element of S_5.
   (b) Section 9, Problem 15: Find the maximum possible order for an element of S_6.
6. **Problem 6.** Section 9, Problem 23(f) (with slight modification): Show that S_n is not cyclic for any $n \geq 3$.
7. **Problem 7.** Section 9, Problem 23(g) (with slight modification): Is A_3 is an abelian group? Prove yes or no.
8. **Problem 8.** Section 9, Problem 29: Show that for every subgroup H of S_n for $n \geq 2$, either all of the permutations in H are even or exactly half of them are even.
9. **Problem 9.** Let G be group. Define the **center** of G, denoted $Z(G)$, to be the following subset:
   $$Z(G) = \{x \in G \mid xy = yx \quad \forall y \in G\}.$$
   (a) Show that $Z(G)$ is a subgroup of G.
   (b) What is the center of D_3?
   (c) What is the center of D_4?
10. **Problem 10.** Let G be a finite group. Prove that there exists a positive integer N such that, for any $x \in G$, $x^N = e$.