Worksheet 4: Symmetric Groups and Cosets

The first page is about S_3; the back is about S_4.

1. Write down all elements of S_3 (cycle notation will be useful for the next problem).

2. Write down all elements of A_3 (remember: there should be 3 elements, and they are the permutations in S_3 that are even).

3. Write down all left cosets of A_3. Write down all right cosets of A_3. Are they the same?

4. (Optional) Want practice with Cayley's Theorem? Cayley's Theorem says $\mathbb{Z}_3$ is isomorphic to a subgroup of S_3. By writing out the isomorphism (remember, it was $\phi(a) = \lambda_a$, where $\lambda_a : \mathbb{Z}_3 \to \mathbb{Z}_3$ is the function $\lambda_a(x) = a + x$) explicitly, find out what subgroup this is. Is this the same subgroup as A_3?

5. Write down all elements of S_4 (cycle notation will be useful for the next problem).

6. Write down all elements of A_4.

7. Write down all elements of D_4 (remember: if we label the vertices of the square $1, 2, 3, 4$, each symmetry gives a permutation).

8. Write down all left cosets of D_4. Write down all right cosets of D_4. Are they the same?