Math 100A: Abstract Algebra I (UC San Diego, fall 2017)  
Homework 3: due November 1 at 5pm

Reminder: no homework due October 25 due to the midterm on October 23.

1. Artin, Chapter 2, exercise 5.5.
2. Artin, Chapter 2, exercise 5.6 for \( n = 2 \) (optional: do the general case).
3. Prove that two elements of \( S_n \) are conjugate if and only if they have the same cycle structure. (This was stated in class, but now I want you to give a detailed proof.)
5. (i) Prove that every finite group of even order contains an element of order 2. (Hint: every element has the same order as its inverse; use this to pair off the elements of the group.)
   (ii) Artin, Chapter 8, exercise 8.3.
6. Artin, Chapter 2, exercise 8.9.
7. Artin, Chapter 2, exercise 10.5.
9. Let \( D_4 \) be the group of permutations of the four vertices of a square that are induced by congruences (this is called the dihedral group of the square).
   (i) Verify that \( D_4 \) has eight elements, and compute a multiplication table.
   (ii) Count the number of elements of each order.
   (iii) Repeat (ii) for the quaternion group (section 2.4). Deduce that there exist (at least) two nonabelian groups of order 8 which are not isomorphic.
10. Challenge problem: for which \( n \) do the permutations \((123), (234), \ldots, ((n-2)(n-1)n)\) generate \( A_n \)? (This came up in office hours and I don’t know the answer!)
11. For fun only: Artin, Chapter 2, exercise M.16.