Note that the due date has been changed back to the usual weekly time. However, I will not have office hours on the 22nd; they will instead be held Monday, November 20, 4–6pm (where as usual, I reserve the right to leave after 5:30 if no one is present).

1) Prove the following statement from lecture: if $G$ is the subgroup of $\mathbb{Z}^+ \times \mathbb{Z}^+$ generated by $(a, b)$ and $(c, d)$, then the index of $G$ is $|ad - bc|$ if this number is nonzero, and otherwise is infinite. (If you get stuck, try the easier statement that $G = \mathbb{Z}^+ \times \mathbb{Z}^+$ if and only if $ad - bc = \pm 1$.)

2) Let $G$ be a discrete group of isometries of the plane with translation group $L$ and point group $\overline{G}$. Suppose that $\overline{G}$ contains an element of order 6 and that $L$ is nontrivial. Prove that $L$ is a hexagonal lattice, i.e., that in some coordinate system (possibly after rescaling) $L$ is generated by $(1, 0)$ and $(\frac{1}{2}, \frac{\sqrt{3}}{2})$.

3) A frieze group is a discrete group of isometries of the plane whose translation group is isomorphic to $\mathbb{Z}^+$. Read the Wikipedia article about frieze groups, then prove that there are exactly 7 frieze groups up to isomorphism.

4) Artin, Chapter 6, exercise 5.3.
5) Artin, Chapter 6, exercise 5.11.
6) Artin, Chapter 6, exercise 6.1(a).
7) Artin, Chapter 6, exercise 6.3.
8) Artin, Chapter 6, exercise 7.1.