(1) Artin, chapter 16, exercise 3.2.
(2) Artin, chapter 16, exercise 4.1.
(3) Artin, chapter 16, exercise 6.1.
(4) Artin, chapter 16, exercise 6.2.
(5) Let $p$ be a prime, and let $k$ be a field of characteristic $p$.
   (a) Let $F$ be the field $k(t)$ (the fraction field of $k[t]$). Prove that the polynomial $x^p - t \in F[x]$ is irreducible. (Hint: since $k[t]$ is a Euclidean domain, Gauss’s lemma and the Eisenstein criterion remain valid using $t$ as the “prime”.)
   (b) Let $K$ be the field $k(x^{1/p}, y^{1/p})$. Prove that $[K : F] = p^2$. (Hint: apply (a) twice.)
   (c) Prove that for any $\alpha \in K$, we have $\alpha^p \in F$. Deduce that there is no primitive element for $K$ over $F$.
(6) Let $P(x) \in \mathbb{C}[x]$ be a polynomial whose coefficients are algebraic over $\mathbb{Q}$. Prove that the roots of $P$ in $\mathbb{C}$ are also algebraic over $\mathbb{Q}$. (Hint: note that the coefficients all belong to some finite extension of $\mathbb{Q}$, over which we can construct a splitting field.)
(7) Let $K$ be the splitting field of $x^4 - 2$ over $\mathbb{Q}$. Prove that $K$ is Galois and $G(K/\mathbb{Q}) \cong D_4$ (the dihedral group of order 8).
(8) Using cyclotomic fields, construct Galois field extensions $K/\mathbb{Q}$ for which:
   (a) $G(K/\mathbb{Q}) \cong C_7$;
   (b) $G(K/\mathbb{Q}) \cong C_4 \times C_8$;
   (c) $G(K/\mathbb{Q}) \cong C_3 \times C_3 \times C_3$.
(9) Put $K = \mathbb{Q}(\zeta_3, 2^{1/3}, \sqrt{5})$. Prove that $K/\mathbb{Q}$ is Galois and $G(K/\mathbb{Q}) \cong S_3 \times C_2$. 