Problem 1. [10 points.]

Give a one-sentence answer to each of the following.

(a) For $\rho : G \to \text{GL}(V)$ an irreducible representation, give the name and statement of the result describing the $G$-invariant transformations $T : V \to V$.

(b) For $F \to K$ a field extension, explain why the minimal polynomial of an element $\alpha \in K$ over $F$ is irreducible.
(c) For $P(x) \in \mathbb{Z}[x]$ for which every root of $P(x)$ in $\mathbb{C}$ has absolute value 1, describe all possible irreducible factors of $P(x)$ over $\mathbb{Q}$.

**Problem 2.** [5 points.]

For $n$ an odd integer, state (with justification) the relationship between the cyclotomic polynomials $\Phi_n(x)$ and $\Phi_{2n}(x)$.

**Problem 3.** [10 points.]

Compute the degrees of the following field extensions. No further justification is needed.

(a) $\mathbb{Q} \to \mathbb{Q}(2^{1/5})$.

(b) $\mathbb{Q} \to \mathbb{Q}(\zeta_{18})$.

(c) $\mathbb{Q}(\sqrt{6}) \to \mathbb{Q}(\sqrt{2} + \sqrt{3})$.

**Problem 4.** [10 points.]

For each of the following, state whether or not the following ruler-and-compass construction is possible or impossible. If possible, do nothing else; if impossible, justify your answer.

(a) Given distinct points $p_0, p_1, p_2$, construct a point $p_3$ such that the measure of $\angle p_0 p_1 p_3$ is one-fourth that of $\angle p_0 p_1 p_2$.

(b) Given the endpoints $p_0, p_1$ of a line segment of length 1, construct a segment of length $2^{1/3}$.

(c) Given points $p_0, p_1$, construct a regular 13-gon with one side equal to $p_0 p_1$.

(d) Given points $p_0, p_1, p_2$ such that the segment $p_0 p_1$ has length 1 and the segment $p_1 p_2$ has length $\cos \frac{2\pi}{7}$, construct a segment of length $\cos \frac{\pi}{7}$.