Homework #6

5.3.7:

(a) $\vec{q}^{(0)} = [1,1]^T$ and $A\vec{q}^{(0)} = [9,-1]^T$ so $s_1 = 9$ and $\vec{q}^{(1)} = [1,-1/9]^T$. We continue this process to get the table:

j	s_j	$(\vec{q}^{(j)})^T$
1	9	$[1, -0.111111111111111]$
2	7.88888888888889	$[1, -0.267605633802817]$
3	7.73239436619718	$[1, -0.293260473588342]$
4	7.70243441266840	$[1, -0.298290834330602]$
5	7.70170916566940	$[1, -0.298413090509221]$
6	7.70158690949078	$[1, -0.298433701718909]$
7	7.70156629828109	$[1, -0.298437176634043]$
7	7.70156282336596	$[1, -0.298437762483838]$
8	7.70156223751616	$[1, -0.298437861254642]$

and the s_j and $\vec{q}^{(j)_2}$ are no longer changing in 6 decimal places.

1.7.10:

(a) $\det[2] = 2 \neq 0$ and

$$\det\begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} = 2 \neq 0$$

$$\det\begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 0 \\ 4 & 1 & -2 \end{bmatrix} = -2 \neq 0$$

$$\det\begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 0 & 0 & 0 \\ 4 & 1 & -2 & 6 \\ -6 & -1 & 2 & -3 \end{bmatrix} = -6 \neq 0.$$

Since none of these determinants are zero, A can be transformed to upper triangular form by operations of type 1 only.

(b) Gaussian elimination:

$$\begin{bmatrix} 2 & 1 & -1 & 3 & 13 \\ -2 & 0 & 0 & 0 & -2 \\ 4 & 1 & -2 & 6 & 24 \\ -6 & -1 & 2 & -3 & -14 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & -1 & 3 & 13 \\ 0 & 1 & -1 & 3 & 11 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & 2 & -1 & 6 & 25 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & -1 & 3 & 13 \\ 0 & 1 & -1 & 3 & 11 \\ 0 & 0 & -1 & 3 & 9 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & -1 & 3 & 13 \\ 0 & 1 & -1 & 3 & 11 \\ 0 & 0 & -1 & 3 & 9 \\ 0 & 0 & 0 & 3 & 12 \end{bmatrix},$$

where multipliers were $m_{21} = -1, m_{31} = 2, m_{41} = -3$ and $m_{32} = -1, m_{42} = 2$ and $m_{43} = -1$.

(c) Back substitution gives

$$\begin{aligned} x_4 &= 4 \\ x_3 &= \frac{9-12}{-1} = 3 \\ x_2 &= \frac{11+3-12}{1} = 2 \\ x_1 &= \frac{13-2+3-12}{2} = 1. \end{aligned}$$

Checking, we see

$$A\vec{x} = \begin{bmatrix} 13 \\ -2 \\ 24 \\ -14 \end{bmatrix} = \vec{b}.$$

1.7.37: Let

$$B = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & & & \\ \vdots & & A^{(1)} & \\ 0 & & & \end{bmatrix}.$$

We show $b_{ij} = b_{ji}$, for $i, j = 2, \dots, n$. Note,

$$b_{ij} = a_{ij} - m_{i1}a_{1j} = a_{ij} - \frac{a_{i1}}{a_{11}}a_{1j}.$$

Using symmetry of A,

$$b_{ji} = a_{ji} - \frac{a_{j1}}{a_{11}}a_{1i} = a_{ij} - \frac{a_{1j}}{a_{11}}a_{i1} = a_{ij} - \frac{a_{i1}}{a_{11}}a_{1j} = b_{ij}.$$

1.7.39: Suppose A is nonsingular and A has LU factorization matrices L and U. Partition

$$\begin{bmatrix} A_k & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = A = LU = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix},$$

where A_k is the kth leading principal submatrix of A. Then $A_k = L_{11}U_{11} + L_{12}U_{21}$. However, L is lower triangular implies $L_{12} = 0$, so $A_k = L_{11}U_{11}$. Then $\det A_k = \det L_{11} \det U_{11}$. But $\det L_{11} = 1$, since L is lower triangular with 1's on the diagonal implies L_{11} is too. Also $\det U_{11} = \prod_{i=1}^{k} u_{ii} \neq 0$, otherwise $\det U = \prod_{i=1}^{n} u_{ii} = 0$ and $\det A = \det L \det U = 0$, making A singular. Thus, $\det A_k = \det U_{11} \neq 0$ and A_k is nonsingular.

Programming:

(a) See "hw6afn.m".

(b) For $N = 1$, we get $s_1 = 1$, with 24 flops; for $N = 10$, we get $s_{10} = 5.56831805483414$, with 240 flops; for $N = 100$, we get $s_{100} = 5.28824561127089$, with 2400 flops.