Math 170A Midterm 1

October 30, 2017

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit**.

Print Name: ______________________________

Student ID: ______________________________

Signature and Date: ______________________________

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

1. (25 pts) Let A be an $n \times n$ symmetric, positive definite, **tridiagonal** matrix. We store and work with A as two vectors: an $n \times 1$ vector y representing the **main diagonal** and an $(n-1) \times 1$ vector w representing the **upper diagonal**. Given the following start for a **Matlab** function:

   function [v,z] = getCholeskyFactor(y,w,n)

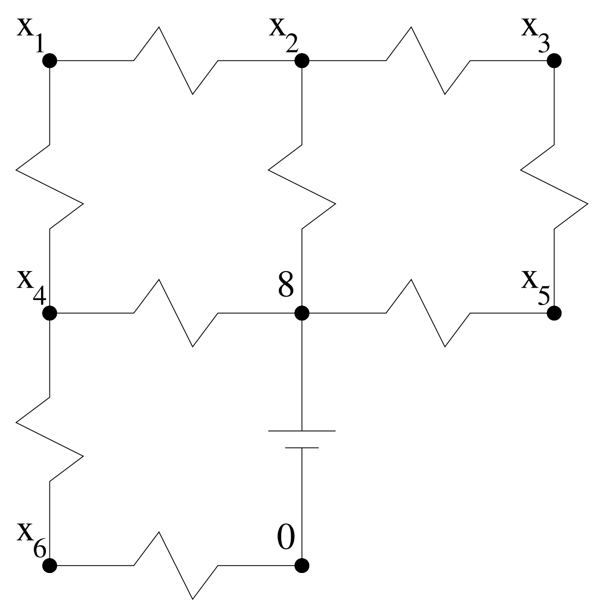
   complete the function so that it uses the **inner product formulation** to compute and output the Cholesky factor's **main diagonal** vector v and **upper diagonal** vector z . Use **only** basic programming and be somewhat efficient in flop counts and comparison (number of "if" statements) counts.

   Remember, R is the Cholesky factor of A means $r_{ij} = 0$ for $i > j$ and $r_{ii} > 0$ and $A = R^T R$. Also, remember inner product formulation formulas:

$$r_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} r_{ki}^2}, \qquad r_{ij} = \left(a_{ij} - \sum_{k=1}^{i-1} r_{ki} r_{kj} \right) \Big/ r_{ii}.$$

2. (25 pts) Consider the **electrical circuit** below, with an 8V battery and resistors each with resistances of 2 Ω. **Write down** a linear system of equations in **matrix form**, $A\vec{x} = \vec{b}$, for the unknown nodal voltages, but do **not** solve.

   Remember Kirchhoff's current law: the sum of currents away from a node must be zero; and Ohm's law: voltage drop is equal to current times resistance.



3. (25 pts) Let

$$R = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

be the **Cholesky factor** of a symmetric, positive definite matrix A. Solve $A\vec{x} = \vec{b}$, when $\vec{b} = [6, -6, 4]^T$, using **column-oriented** forward substitution and **row-oriented** back substitution.

Remember row-oriented back substitution: $x_i = \left(b_i - \sum_{i=i+1}^{n} g_{ij} x_j\right) / g_{ii}$.

4. (25 pts) Let $A = (a_{ij})$ be a 20×20 symmetric, positive definite matrix, and suppose $a_{i,15} = 0$ for all $1 \leq i \leq 14$. If $R = (r_{ij})$ is the **Cholesky factor** of A, prove $r_{i,15} = 0$ for all $1 \leq i \leq 14$.