# Math 170A Midterm

February 5, 2014

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit.**

Print Name: ________________________________________________

Student ID: ________________________________________________

Signature and Date: _________________________________________

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<th>Problem</th>
<th>Score</th>
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<tr>
<td>1</td>
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1. (25 pts) Given the following header for a Matlab function:

   function [x] = ColumnOrientedBackSub(A,b,n)

   Complete the function so that it performs column-oriented back substitution to solve the linear system $Ax = b$ given $A$ an $n \times n$, upper triangular matrix. Use only basic programming (do not use Matlab’s in-built vector-vector addition, matrix-vector multiplication, matrix-matrix multiplication, or linear system solver).
2. (25 pts) Let $A = (a_{ij})$ be a $10 \times 10$, positive definite matrix with Cholesky factor $R = (r_{ij})$. Suppose that $a_{1,1} = 16$, $a_{1,2} = -8$, $a_{1,7} = 12$, $a_{2,2} = 13$, and $a_{2,7} = -21$. Solve for the value of $r_{2,7}$. 
3. (25 pts) Consider an \( m \times m \) network of nodes, such as in the following figure for \( m = 5 \), with one equation, one unknown at each node:

Suppose the \( i \)th equation is linear and involves only the unknowns associated with the \( i \)th node and the nodes in the immediate up, down, left, right, and diagonal directions (for example, the figure’s 13th equation involves \( x_7, x_8, x_9, x_{12}, x_{13}, x_{14}, x_{17}, x_{18}, x_{19} \)). Answer the following questions for general \( m \):

(a) Count, exactly, the maximum number of nonzeros possible, in terms of \( m \), in the system’s matrix.

(b) Find the bandwidth of the matrix in terms of \( m \).
4. (25 pts) Suppose $A$ is an $n \times n$, positive definite matrix and suppose we partition it into $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, where $A_{11}$ is of size $k \times k$, for $0 < k < n$. Prove $A_{11}$ is also positive definite.