Math 170A Midterm 2

November 20, 2017

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit**.

Print Name: ____

Student ID: ____

Seat Number: __________

Signature and Date: ____

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

1. (25 pts) Given the following start for a **Matlab** function:

   function [s] = PowerMethod(n,A,b,N)

   that inputs

   - dimension n;
   - $n \times n$ matrix A and $n \times 1$ vector b;
   - number of steps N;

   complete the function so that it performs N steps of the **Power method**, using **initial guess**, $\vec{q}^{(0)}$, a vector of all ones, and outputs the approximate **eigenvalue** s_N. Do **not** use Matlab's in-built constant-vector, vector-vector, or matrix-vector multiplications.

   Remember Power method iterations: $\vec{q}^{(k+1)} = \frac{A\vec{q}^{(k)}}{s_{k+1}}$, where $s_{k+1} = (A\vec{q}^{(k)})_j$ such that $|(A\vec{q}^{(k)})_j| = \max_{1 \le i \le n} |(A\vec{q}^{(k)})_i|$.

2. (25 pts) Find the L and U matrices for the **LU factorization** of

$$A = \begin{bmatrix} 2 & -1 & 6 \\ 1 & 1 & 0 \\ -4 & -2 & 2 \end{bmatrix}.$$

Remember multiplier: $m_{ij} = \frac{a_{ij}}{a_{jj}}$.

3. (25 pts) Consider

$$A = \begin{bmatrix} -1 & c \\ c & 2 \end{bmatrix}.$$

Find **all** c such that **Gauss-Seidel** on $A\vec{x} = \vec{b}$ will **converge** for any initial guess.

Remember Gauss-Seidel iterations: $\vec{x}^{(k+1)} = (D - E)^{-1}(F\vec{x}^{(k)} + \vec{b})$, where $A = D - E - F$, and D is the diagonal, $-E$ the strictly lower triangular, and $-F$ the strictly upper triangular portions of A.

Also,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}.$$

4. (25 pts) Given n, define the **norm** $||\cdot||$ by

$$||\vec{x}|| = 2 \max_{1\le i\le n} |x_i|.$$

Find $C_1, C_2, C_3 > 0$ such that

$$C_1||\vec{x}||_1 \le ||\vec{x}|| \le C_2||\vec{x}||_1,$$

and

$$||A|| \le C_3||A||_1,$$

for all $n \times 1$ vectors $\vec{x}$ and $n \times n$ matrices A.

Remember:

$$||\vec{x}||_1 = \sum_{i=1}^{n} |x_i|, \quad ||A|| = \max_{\vec{x}\neq 0} \frac{||A\vec{x}||}{||\vec{x}||}, \quad ||A||_1 = \max_{\vec{x}\neq 0} \frac{||A\vec{x}||_1}{||\vec{x}||_1}.$$