Homework #1 Sketch

1. For each part, check whether the hypotheses of the intermediate value theorem are satisfied for the starting interval \([a, b]\): \(f\) continuous and \(f\) has different signs at \(a\) and \(b\).

2. Perform bisection method until only one root, among 0, 1, \(-2\), is in the interval. The bisection method will converge to this root.

3. (a) Use \(|r - c_n| \leq (b - a) / 2^{n+1}\) for a bound.
   (b) Set the bound \((b - a) / 2^{n+1}\) to be less than \(10^{-11}\) and solve for \(n\).

4. (a) Bisection method should use interval \([0, b_n]\) for all \(n\). Starting, \(b_0 = b\), and the interval is halved every iteration, so \(b_1 = b/2, b_2 = b/4, \ldots\), and you can find the general formula for \(b_n\) in terms of \(b\).
   (b) Convergence is to 0 and \(|e_n|\) will be similar in form to \(C/2^n\), which is linearly convergent, so show \(|e_{n+1}|/|e_n|\) converges to a number < 1, which means it is bounded.

5. (a) \(c_0 = (a_0 + b_0)/2\) and \(c_1\) is either \((a_0 + c_0)/2\) or \((c_0 + b_0)/2\). Note one is \(> c_0\) and one is \(< c_0\). For each case, you have two equations, two unknowns that you can use to solve for \(a_0\) and \(b_0\).
   (b) Find out which case you are in and use your results in the previous part.

6. (a) For each variation, bound the interval length \(b_1 - a_1, b_2 - a_2, \ldots\), in terms of \(b - a\), using the cut location. Finally bound the approximation for the final interval \([a_n, b_n]\).
   (b) Follow the procedures of each variation to calculate the approximations.

7. (a) Calculate approximations using bisection method.
   (b) Compute the bound for bisection method using \((b - a)/2^{n+1}\) and compute the absolute errors.
   (c) Calculate approximations using Newton’s method and compute absolute errors and compare them to those in the previous part.

8. (a) True
   (b) False
   (c) False
   (d) False
   (e) True
   (f) False

9. (Matlab) See Matlab solutions.