Homework #3

1. Compute using Newton’s method on 2 equations, 2 unknowns.

2. (a) Put all terms in both expressions under the same denominator and simplify the numerator to show they are equal.
   (b) No.

3. Compute using secant method.

4. (a) Write $|e_{n+2}|$ in terms of $|e_{n+1}|$ and $|e_{n+1}|$ in terms of $|e_n|$ and put it all together.
   (b) Secant method is faster.

5. (a) True
   (b) True
   (c) False
   (d) False
   (e) True
   (f) True

6. For each part, compute absolute maximum $M$ and absolute minimum $m$ of $F$ in the interval $[a,b]$ by comparing values at critical points and endpoints. Then use the fact that $F : [a, b] \to [m, M]$ and $F(c) = m, F(d) = M$ for some $c, d \in [a, b]$.

7. (a) Use the same technique as in the previous problem.
   (b) Using a similar technique as in the previous part and problem, check the value of $|F'|$ at critical points of $F'$ and endpoints to find the global maximum. of $|F'|$ in the interval.
   (c) Use the result $|x_n - s| \leq \lambda^n|x_0 - s|$, where $s$ is the fixed point, which comes from mean value theorem.
   (d) Compute the value and absolute error. The absolute error will be very small because the method is actually Newton’s method and quadratically convergent, while the previous part assumed the worst case of linear convergence with asymptotic error constant $\lambda$.

8. $F'$ exists at $s$ implies it is continuous in a neighborhood around $s$. In this neighborhood, the continuity implies given $\epsilon > 0$, there exists a subneighborhood such that $|F'(x)| < |F'(s)| + \epsilon$. Choosing $\epsilon$ properly, we can find the desired $\delta > 0$. Then for $|x - s| \leq \delta$, show $|F(x) - s| \leq \delta$. These give you the hypotheses needed in the theorem for convergence of fixed point iterations.

9. (Matlab) See Matlab solutions.