Homework #4

1. (a) Suppose \( f(1) = 4, \ f(2) = 3, \) and \( f(3) = 1. \) Find the Lagrange form of the Lagrange interpolating polynomial. Then simplify it to the form \( p(x) = ax^2 + bx + c. \)

(b) Use the quadratic formula to find the roots of \( p(x). \)

(c) Starting with initial guesses at \( p_0 = 1, \ p_1 = 2, \ p_2 = 3, \) what is the next approximation, \( p_3, \) to the root of \( f(x) \) using Muller’s method?

(d) How many function evaluations are needed to generate \( p_N? \)

2. Given \( n + 1 \) data points with distinct nodes, prove there are an infinite number of different polynomials of degree > \( n \) interpolating these data points (Hint: add a data point).

3. (a) Find the Lagrange form of the Lagrange interpolating polynomial for the data points \( (x_0 - h, f(x_0 - h)), (x_0, f(x_0)), (x_0 + h, f(x_0 + h)) \) and call it \( p_2(x). \)

(b) Simplify \( p_2'(x_0). \)

(c) Simplify \( p_2''(x_0). \)

(d) (not due) Simplify \( \int_{x_0-h}^{x_0+h} p_2(x) \ dx. \)

4. (Matlab) Write a Matlab program that inputs

- an integer \( m; \)
- two vectors of data points \( x \) and \( y, \) both with \( m \) components;
- a single point \( z. \)

Have this program form the Lagrange form for the interpolation polynomial and output its value at \( z. \)

(a) Write out or print out your program.

(b) Apply your program to the case with data points \((-1,3), (0,1), (1,-1), (2,3)\) and write out or print out your results for \( z = -2, -0.5, 0.1, 1.7, 3. \)

5. (not due) Explain why the unique polynomial of degree \( \leq n \) that interpolates \( n + 1 \) data points is also the polynomial of least degree that interpolates those data points.

6. (not due) Consider the data points \((-2,-1), (0,1), (-1,3). \)

(a) Write down the Lagrange form for the least degree polynomial \( p_2(x) \) interpolating these data points.

(b) Find \( K, a, b, c \) such that \( p_3(x) = p_2(x) + K(x-a)(x-b)(x-c) \) interpolates, in addition, the data point \((1,-1).\)

(c) Evaluate \( p_3(0.7) \) and \( p_3'(0.7). \)
7. (Matlab) (not due) Write a Matlab function that inputs

- $a, b$, endpoints of a starting interval;
- $tol$, a tolerance;

and outputs the approximation $c_N$ of bisection method satisfying $(b - a)/2^{N+1} \leq tol$.

Now suppose we know that $f(x) = x^5 + 3x^4 - 8x^3 - 12x^2 + 16x$ has integer roots. Use the method of deflation to get all 5 roots of $f(x)$ by using your bisection method, along with the method of deflation, using starting interval endpoints either $a = -6, b = 3$ or $a = -6, 1.5$, whichever has difference of sign in $f(x)$, and $tol$ appropriately chosen (so only one integer lies in the final interval).