Homework #4 Sketch

1. Suppose there is a polynomial of smaller degree that interpolates the data. Then its degree is still \( \leq n \). Then use uniqueness of interpolation polynomial to arrive at a contradiction.

2. Fix \( x_{n+1} \) to be a node that is distinct from \( x_0, \ldots, x_n \) and study the addition of the data point \((x_{n+1}, y_{n+1})\). Argue that each different chosen \( y_{n+1} \) leads to a different interpolation polynomial. These will form an infinite number of polynomials that interpolate the original data points. Finally argue that all but one of these have degree \( > n \) due to uniqueness of interpolation polynomial.

3. Use formula of Lagrange form and iterative construction of Newton form.

4. (a) Use formula for Lagrange form.
   (b) Choose \( a, b, c \) to be at the nodes of the first 3 data points. Then find \( K \) by satisfying \( p_3(1) = -1 \).
   (c) Plug in \( x = 1/2 \) into your expression for \( p_3(x) \).

5. (Matlab) See Matlab solutions.