Homework #5

1. Consider the data points \((-2, -1), (0, 1), (-1, 3)\).
   
   (a) Write down the Newton form for the interpolation polynomial for this data.
   (b) Add the data point \((1, -1)\) and write down the new Newton form.
   (c) Evaluate the polynomial of part (b) and its derivative at \(x = 1/2\).

2. (a) Simplify \(f[x_0, x_1, x_2]\) and \(f[x_2, x_0, x_1]\) and verify they are the same.
   (b) Also verify that the Newton forms for the interpolation polynomials using the data points with nodes \(x_0 = a, x_1 = b\) and the data points with nodes in reverse order \(x_0 = b, x_1 = a\) simplify to the same polynomial.

3. Suppose \(x_0 = 0, x_1 = 0.4, x_2 = 0.7\) and \(f[x_2] = 6, f[x_1, x_2] = 10, f[x_0, x_1, x_2] = 50/7\). Find the values of \(f[x_0], f[x_1], f[x_0, x_1]\).

4. Consider the data points \((-2, 1), (-1, 4), (0, 11), (1, 16), (2, 13), (3, -4)\).
   
   (a) Write down the divided difference table associated to this data.
   (b) Determine from the table the degree of the interpolation polynomial passing through these data points.

5. Use calculus to find the constant \(C\) such that

\[ |f(x) - P(x)| \leq C \]

for all \(x \in [0, 0.3]\), where \(P(x)\) is the interpolation polynomial for data with nodes \(x_0 = 0, x_1 = 0.1, x_2 = 0.3\) and values from the underlying function \(f(x) = e^{x^2+1}\).

6. Use calculus to find the constant \(C\) such that

\[ |f(x) - P(x)| \leq C \]

for all \(x \in [0, 1]\), where \(P(x)\) is the piecewise linear interpolating polynomial for data with nodes \(x_j = j/10, j = 0, \ldots, 10\) and values from the underlying function \(f(x) = x^2 + 1\).

7. Find \(n\) such that the piecewise linear interpolating polynomial for data with nodes \(x_j = 2j/n, j = 0, \ldots, n\) and values from the underlying function \(f(x) = \sin x\) has absolute error in \([0, 2]\) less than \(10^{-5}\).

8. (Matlab) Write a Matlab program that inputs:
   
   - \(n\);
   - vectors \(x\) and \(y\), the \(x\) and \(y\) coordinates of \(n + 1\) data points;
and outputs the value of the $n$th divided difference.

(a) Write out or print out your program.

(b) Apply your program to the data points of problem #1(b) and #4. Write out or print out your answer.