Homework #5

1. Consider the data points \((-2, -1), (0, 1), (-1, 3)\).
   (a) Write down the Newton form for the interpolation polynomial for this data.
   (b) Add the data point \((1, -1)\) and write down the new Newton form.
   (c) Evaluate the polynomial of part (b) and its derivative at \(x = 1/2\).

2. (a) Find the interpolation polynomial for the data points \((x_0-h, f(x_0-h)), (x_0, f(x_0)), (x_0+h, f(x_0+h))\) and call it \(p_2(x)\).
   (b) Simplify \(p_2'(x_0)\).
   (c) Simplify \(p_2''(x_0)\).
   (d) Simplify \(\int_{x_0-h}^{x_0+h} p_2(x) \, dx\).

3. Use calculus to find the constant \(C\) such that
   \[ |f(x) - P(x)| \leq C \]
   for all \(x \in [0, 0.3]\), where \(P(x)\) is the interpolation polynomial for data with nodes \(x_0 = 0, x_1 = 0.1, x_2 = 0.3\) and values from the underlying function \(f(x) = e^{x+1}\).

4. Use calculus to find the constant \(C\) such that
   \[ |f(x) - P(x)| \leq C \]
   for all \(x \in [0, 1]\), where \(P(x)\) is the piecewise linear interpolating polynomial for data with nodes \(x_j = j/10, j = 0, \ldots, 10\) and values from the underlying function \(f(x) = x^2 + 1\).

5. Find \(n\) such that the piecewise linear interpolating polynomial for data with nodes \(x_j = 2j/n, j = 0, \ldots, n\) and values from the underlying function \(f(x) = \sin x\) has absolute error in \([0, 2]\) less than \(10^{-5}\).

6. Use nodes at the zeros of the degree three Chebyshev polynomial to construct an interpolating polynomial of degree \(\leq 2\) for the function \(f(x) = e^x\) in the interval \((-1, 1)\). Also, bound the maximum error of the approximation in \((-1, 1)\):

7. Use nodes at the zeros of the degree four Chebyshev polynomial to construct an interpolating polynomial of degree \(\leq 3\) for the function \(f(x) = \sin x\) in the interval \((-1, 1)\). Also, bound the maximum error of the approximation in \((-1, 1)\):

8. (a) Simplify \(f[x_0, x_1, x_2]\) and \(f[x_2, x_0, x_1]\) and verify they are the same.
   (b) Also verify that the Newton forms for the interpolation polynomials using the data points with nodes \(x_0 = a, x_1 = b\) and the data points with nodes in reverse order \(x_0 = b, x_1 = a\) simplify to the same polynomial.
9. (Matlab) Write a Matlab program that inputs $x$ and outputs $f(x)$. Then write a Matlab program that inputs: location $x$ and stepsize $h$, and outputs the result of your formula of #2(c).

(a) Write out or print out your program.
(b) Apply your program to approximate $f''(0)$, where $f(x) = x^2$, using $h = 0.1$. What is the absolute error?
(c) Apply your program to approximate $f''(0)$, where $f(x) = \sin x$, using $h = 0.1, 0.05$. What are the absolute errors?