Homework #5

1. Consider the data points \((-2, -1), (0, 1), (-1, 3)\).
   (a) \(p_2(x) = -1 + (x + 2) - 3(x + 2)x\).
   (b) \(p_3(x) = -1 + (x + 2) - 3(x + 2)x + (x + 2)x(x + 1)\).
   (c) \(p_3(1/2) = -3/8\) and \(p'_3(1/2) = -9/4\).

2. (a) Simplify the expressions
   \[
   f[x_0, x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0},
   \]
   \[
   f[x_2, x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} - \frac{f(x_0) - f(x_2)}{x_0 - x_2}.
   \]

   (b) Simplify the expressions \(f(a) + \frac{f(b) - f(a)}{b-a} (x - a)\) and \(f(b) + \frac{f(a) - f(b)}{a-b} (x - b)\).

3. \(f[x_1] = 3\) and \(f[x_0, x_1] = 5\) and \(f[x_0] = 1\).

4. (a) First divided differences: \([3; 7; 5; -3; -17]\); second divided differences: \([2; -1; -4; -7]\);
   third divided differences: \([-1; -1; -1]\); fourth divided differences: \([0; 0]\); fifth divided difference: \(0\).
   (b) Degree will be exactly 3.

5. Use bound
   \[
   |f(x) - P(x)| \leq \frac{\max_{y \in [0, 0.3]} |f'''(y)|}{6} \max_{z \in [x_0, x_2]} |(z - x_0)(z - x_1)(z - x_2)|,
   \]
   and use calculus, studying critical points and endpoints, to get
   \[\max_{y \in [0, 0.3]} |f'''(y)| = e^{1.3}\]
   and
   \[\max_{z \in [x_0, x_2]} |(z - x_0)(z - x_1)(z - x_2)| = 0.00211261179092238.\]
   So
   \[|f(x) - P(x)| \leq 0.00129196656740077.\]

6. Use bound
   \[
   |f(x) - P(x)| \leq \frac{|f''(\xi(x))|}{2} \max_{z \in [x_i, x_{i+1}]} |(z - x_i)(z - x_{i+1})|,
   \]
where $x \in [x_i, x_{i+1}]$, and use $f''(\xi(x)) = 2$ and calculus, studying critical points and endpoints, to get

$$\max_{z \in [x_i, x_{i+1}]} |(z - x_i)(z - x_{i+1})| = \frac{h^2}{4} = \frac{1}{400}.$$ 

So

$$|f(x) - P(x)| \leq \frac{1}{400}.$$ 

7. Use

$$|f(x) - P(x)| \leq \max_{y \in [0, 2]} |f''(y)| \max_{z \in [x_i, x_{i+1}]} |(z - x_i)(z - x_{i+1})| = \frac{h^2}{8}$$

and force $h^2/8 \leq 10^{-5}$ to solve for $n$, since $h = 2/n$. This gives $n \geq 223.606797749979$.

8. (Matlab) See online Matlab solutions.