1. Interpolating values at \( x_i, \ i = 0, 1, 2, 3 \) is 4 conditions, derivatives at \( x_i, \ i = 0, 1, 2 \) is 3 more, 2nd derivatives at \( x_i, \ i = 0, 1 \) is 2 more, and 3rd derivative at \( x_0 \) is 1 more, for a total of 10. Equating number number of degrees of freedom with number of conditions, \( d + 1 = 10 \), so \( d = 9 \) conditions

2. (a) Verify, noting lots of simplification when evaluating at nodes.  
(b) Verify, noting lots of simplification when evaluating at nodes.

3. (a) Note that \( \text{deg} \ p = 3 \) and verify that \( p(0) = -2, p'(0) = -1, p(1) = 1, p'(1) = 8 \).
(b) Try \( q(x) = p(x) + Cx^2(x - 1)^2 + Kx^2(x - 1)^2(x - 2) \) and enforce \( q(2) = 0 \) and \( q'(2) = 0 \) to get \( C = -4 \) and \( K = 25/4 \).

4. (a) \( p(x) = (x + 1) - \frac{1}{2}(x + 1)^2(x - 1) \)
(b) \( p(x) = (x + 1) - \frac{1}{2}(x + 1)^2(x - 1) - \frac{1}{2}(x + 1)^2(x - 1)^2 + \frac{1}{2}(x + 1)^2(x - 1)^2x \).
(c) \( p(x) = (x + 1) + (x + 1)^2x \) in \([-1, 0]\) and \( = 1 + 2x - x^2 - x^2(x - 1) \) in \([0, 1]\).

5. (a) Use bound
\[
|f(x) - p(x)| \leq \frac{\max_{y \in [x_i, x_{i+1}]} |f^{(4)}(y)|}{4!} |(1 - x_i)^2(1 - x_{i+1})^2|
\]
where \( 1 \in [x_i, x_{i+1}] \), so \( i = 1 \) and \( x_i = \pi/5 \) and \( x_{i+1} = 2\pi/5 \). So
\[
|f(x) - p(x)| \leq 3.60558426708222 \cdot 10^{-4}.
\]
(b) Use
\[
|f(x) - p(x)| \leq \frac{h^4}{384}
\]
and force \( h^4/384 \leq 10^{-10} \), so \( n \geq 448.845693100611 \).

6. Form
\[
g(t) = f(t) - p(t) - (f(x) - p(x)) \prod_{i=0}^n (t - x_i)^2 \prod_{i=0}^n (x - x_i)^2
\]
so that \( g \) has roots at \( x_i \), for all \( i \), and \( x \). Check that \( g' \) has roots at \( x_i \), for all \( i \). Using Rolle’s theorem, conclude that \( g' \) has at least \( 2n + 2 \) roots. So Generalized Rolle’s theorem on \( g' \) says there is a \( \xi_x \in (a, b) \) such that \( (g')^{(2n+1)}(\xi_x) = 0 \). Simplify the expression for \( g^{(2n+2)}(\xi_x) \) to get the desired result, using what you know about \( m \) and \( m + 1 \) derivatives of polynomials of degree \( \leq m \).

7. (a) Number of degrees of freedom: \((k + 1)n\).
(b) Number of conditions: \(2n\).
(c) Number of conditions: \(k(n - 1)\).
(d) \(6n - (2n + 4(n - 1)) = 4\) degrees of freedom remaining.

8. (Matlab) See online Matlab solutions.