Homework #6 Sketch

1. Form the divided difference table with the given values and fill in the rest of it for the unknown values using the recursive formula for calculating divided differences.

2. (a) Form the divided difference table.
   (b) Add the data point to the top (or bottom) of your divided difference table.
   (c) Check the values of $f[-2, -1, 0, 1, 2, 3], f[-2, -1, 0, 1, 2], f[-2, -1, 0, 1], f[-2, -1, 0], f[-2, -1], f[-2]$ in order. The first nonzero one will review the degree.

3. Simplify the expressions. There will be lots of zeros or cancellations when you plug in the location $x_k$.

4. (a) Check that $p(x_0) = f(x_0), p'(x_0) = f'(x_0), p(x_1) = f(x_1), p'(x_1) = f'(x_1)$.
   (b) Solve for $C, D$ in the Newton form
   
   $$p(x) + C(x - x_0)^2(x - x_1)^2 + D(x - x_0)^2(x - x_1)^2(x - x_2).$$

5. Given $x_0 = -1, x_1 = 1$ and $f(x_0) = 0, f(x_1) = 2$ and $f'(x_0) = 1, f'(x_1) = -1$:
   (a) Use the doubling of nodes idea to get the divided difference table and the corresponding polynomial.
   (b) Add the double of $x_2$ to the top (or bottom) of your divided difference table and get the polynomial associated to it.
   (c) Compute the form of
   
   $$p(x) = \begin{cases} 
   q(x), & x \in [-1, 0] \\
   r(x), & x \in [0, 1] 
   \end{cases}$$

   where $q(x), r(x)$ are cubic polynomials satisfying

   $$q(-1) = f(-1), q'(-1) = f'(-1), q(0) = f(0), q'(0) = f'(0)$$
   $$r(0) = f(0), r'(0) = f'(0), r(1) = f(1), r'(1) = f'(1).$$

   You can do this by extracting the Newton forms for $q$ and $r$ from your already computed divided difference table in the previous part (if you ordered the nodes $-1, 0, 1$; otherwise, you can get one of either $q$ or $r$).

6. (Matlab) See Matlab solutions.