Homework #9

1. (a) We find a cubic polynomial $q$ satisfying $||p - q|| \leq ||p - h||$, for all cubic polynomials $h$. For any cubic polynomial $h$, $(p - h)/3$ is monic, and

$$||(p - h)/3|| \geq ||T_4/8||,$$

where $T_4(x)/8$ is the monic Chebyshev polynomial. Thus, choosing $q$ satisfying

$$p(x) - q(x) = T_4(x),$$

we get $||(p - h)/3|| \geq ||(p - q)/3||$, for all cubic polynomials $h$, and so

$$||p - h|| \geq ||p - q||.$$

Simplifying,

$$q(x) = p(x) - \frac{3}{8} T_4(x) = 3x^4 - 2x^2 + x - 1 - (3x^4 - 3x^2 + 3/8) = x^2 + x - 11/8.$$

2. The polynomial is

$$p(x, y) = \sum_{i=1}^{3} \sum_{j=1}^{3} f(x_i, y_j)u_i(x)v_j(y),$$

where

$$u_i(x) = \prod_{k=1, k \neq i}^{3} \frac{x - x_k}{x_i - x_k},$$

and

$$v_j(y) = \prod_{k=1, k \neq j}^{3} \frac{y - y_k}{y_i - y_k}.$$  

Note $u_i(x)v_j(y)$ is a cardinal function in the sense $u_i(x_r)v_j(y_s) = 1$, when $r = i$ and $s = j$, and = 0 for all other $x_r, y_s$. It is also a product of cardinal functions $u_i(x)$ and $v_j(y)$, noting $u_i(x_i) = 1$ and = 0 for other nodes, and $v_j(y_j) = 1$ and = 0 for other nodes.

We get

$$u_1(1.7) = \frac{(1.7 - 1)(1.7 - 3)}{(0 - 1)(0 - 3)} = -0.30333,$$
$$u_2(1.7) = \frac{(1.7 - 0)(1.7 - 3)}{(1 - 0)(1 - 3)} = 1.105,$$
$$u_3(1.7) = \frac{(1.7 - 0)(1.7 - 1)}{(3 - 0)(3 - 1)} = 0.595,$$
$$v_1(0.2) = \frac{(0.2 - 1)(0.2 - 2)}{(-1 - 1)(-1 - 2)} = 0.24,$$
$$v_2(0.2) = \frac{(0.2 + 1)(0.2 - 2)}{(1 + 1)(1 - 2)} = 1.08,$$
$$v_3(0.2) = \frac{(0.2 + 1)(0.2 - 1)}{(2 + 1)(2 - 1)} = -0.32.$$
So

\[
p(1.7, 0.2) = \sqrt{1}(-0.30333)(0.24) + \sqrt{1}(-0.30333)(1.08) + \sqrt{4}(-0.30333)(-0.32) + \\
\sqrt{2}(1.105)(0.24) + \sqrt{2}(1.105)(1.08) + \sqrt{5}(1.105)(-0.32) + \\
\sqrt{10}(0.595)(0.24) + \sqrt{10}(0.595)(1.08) + \sqrt{13}(0.595)(-0.32) \\
= 3.9187.
\]

3. (Matlab)

(a) See “hw9afn.m”.

(b) Using the program, the value at (0.231, 0.534) is 0.58197; the value at (0.612, 0.311) is 0.68675; and the value at (0.892, 0.789) is 1.191.