Math 170B Midterm 2
February 29, 2012

• Please put your name, ID number, and sign and date.
• There are 4 problems worth a total of 100 points.
• You must show your work to receive credit.

Print Name: ____________________________

Student ID: ____________________________

Signature and Date: ______________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
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<tr>
<td>1</td>
<td>/25</td>
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<td>2</td>
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<td>3</td>
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<td>/25</td>
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<td>Total</td>
<td>/100</td>
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</tbody>
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1. (25 pts) Write a Matlab function that inputs \( m \); two vectors of data points \( x \) and \( y \), both with \( m \) components; and a location \( z \). Have this program use the Lagrange form to output the value of the polynomial of least degree interpolating the data points at the location \( z \).
2. (25 pts) Show there is at most one polynomial of degree $\leq n$ that interpolates $n + 1$ data points with distinct nodes, for $n \geq 0$. If you use the Fundamental Theorem of Algebra, state the version you are using.
3. (25 pts) Circle **TRUE** or **FALSE** for the following questions. You do not need to show your work just for this problem.

(a) There exists 5 data points with distinct nodes such that one can find a degree 4 polynomial and a degree 2 polynomial that each interpolate the data points.

**TRUE** or **FALSE**

(b) Let \( p \) be the polynomial of least degree interpolating the data points \((x_0, f(x_0)), \ldots, (x_n, f(x_n))\) with distinct nodes. If \( f[x_0, \ldots, x_k] \neq 0 \), for some \( 0 \leq k \leq n \), and \( f[x_0, \ldots, x_j] = 0 \), for \( k < j \leq n \), then \( \deg p = k \).

**TRUE** or **FALSE**

(c) Given distinct nodes \( x_0, \ldots, x_n \) in the interval \([a, b]\), let \( g_i(x) \) be a possibly non-polynomial function defined over \([a, b]\) such that

\[
g_i(x_j) = \begin{cases} 
0, & \text{if } i \neq j \\
1, & \text{if } i = j.
\end{cases}
\]

Then the possibly non-polynomial function \( f(x) = \sum_{i=0}^{n} y_i g_i(x) \) interpolates the data \((x_0, y_0), \ldots, (x_n, y_n)\).

**TRUE** or **FALSE**

(d) There exists an \( n \geq 0 \) along with \( n + 1 \) data points with distinct nodes and a \( k > n \) such that no polynomial of degree \( k \) interpolates the data points.

**TRUE** or **FALSE**

(e) Let \( z_1, \ldots, z_n \), with \( n \geq 1 \), be the \( n \) roots of the degree \( n \) Chebyshev polynomial \( T_n(x) \). Then \( T_n(x) = 2^n - 1 \prod_{i=1}^{n} (x - z_i) \).

**TRUE** or **FALSE**

(f) \( p(x) = x^4 - x^3 + x^2 - x + 1 \) is the polynomial of least degree interpolating the data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>1</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>( y' )</td>
<td>-10</td>
<td>-1</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

**TRUE** or **FALSE**
4. (25 pts)

(a) Find the Lagrange form of the polynomial of least degree interpolating values of \( f(x) = x^4 \) at the three roots of the degree 3 Chebyshev polynomial.

(b) Let \( p_n \) be the piecewise linear polynomial interpolating the values of \( f(x) = e^{4x} \) in \([-1, 1]\) using the equally spaced nodes \( x_i = -1 + 2i/n, \ i = 0, \ldots, n \). Use the error bound of polynomial interpolation to find \( n \) ensuring \(|f(x) - p_n(x)| \leq 10^{-12}\).
(c) Find the polynomial of least degree satisfying $p(0) = -1$ and $p(1) = 2$ and $p'(1) = -1$ and $p''(1) = 4$.

(d) Approximate the value at $x = 3$ using the piecewise linear interpolating polynomial for the data:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>