Math 170B Midterm 2
February 24, 2016

• Please put your name, ID number, and sign and date.
• There are 4 problems worth a total of 100 points.
• You must show your work to receive credit.

Print Name: ________________________________

Student ID: ________________________________

Signature and Date: _________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/25</td>
</tr>
<tr>
<td>2</td>
<td>/25</td>
</tr>
<tr>
<td>3</td>
<td>/25</td>
</tr>
<tr>
<td>4</td>
<td>/25</td>
</tr>
<tr>
<td>Total</td>
<td>/100</td>
</tr>
</tbody>
</table>
1. (25 pts) (Matlab) Write a Matlab program that takes as input:
   
   - $m$, the number of nodes;
   - $x$, a vector of nodes;
   - $y$, a vector of values at the nodes;
   - $yprime$, a vector of first derivative values at the nodes;

   and outputs the coefficient of $x^{2m-1}$, for the Hermite interpolation polynomial, using divided differences.
2. (25 pts) Let \( f(x) = e^{x-1} \). For a choice of \( n + 1 \) nodes, \( x_0, \ldots, x_n \in [-1, 1] \), let \( p(x) \) be the interpolation polynomial for the data points \( (x_0, f(x_0)), \ldots, (x_n, f(x_n)) \). Find one example of \( n \), and node locations \( x_0, \ldots, x_n \), satisfying \( |f(x) - p(x)| \leq \frac{1}{1000} \) for all \( x \in [-1, 1] \). You must justify your answer.

Hint: Use Chebyshev polynomials, error bounds, and check \( n = 1, 2, 3, \ldots \) Remember:

\[
T_k(x) = \cos(k \cos^{-1}(x)), \quad T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x)
\]

\[
T_k \left( \cos \left( \frac{(2j-1)x}{2k} \right) \right) = 0, \quad \max_{x \in [-1, 1]} |T_k(x)| = 1
\]

and for \( x \in [-1, 1] \), and for some \( \xi_x \in [-1, 1] \),

\[
f(x) - p(x) = \frac{f^{(n+1)}(\xi_x)}{(n+1)!}(x-x_0) \cdot \cdots \cdot (x-x_n).
\]
3. (25 pts) Let \( f(x) \) be a given function and suppose \( p_6(x) = x^3 - 5x^2 + 18 \) is the interpolation polynomial for the table of data:

\[
\begin{array}{c|ccccccc}
 x & -4 & -3 & -2 & -1 & 1 & 3 & 4 \\
 y & f(-4) & f(-3) & f(-2) & f(-1) & f(1) & f(3) & f(4)
\end{array}
\]

Let \( p_7(x) \) be the interpolation polynomial that adds the data point with node at \( x = 0 \) with value \( f(0) = 10 \). Evaluate \( p_7(2) \) and simplify it to be an integer.
4. (25 pts) Let \( F(x) = \frac{1}{3 - x} + \frac{x}{2} \). Prove fixed point iterations, \( x_{n+1} = F(x_n) \), will converge for all initial guesses in \( \left[ \frac{1}{2}, \frac{3}{2} \right] \).

Remember: MVT says \( F(x) - F(y) = F'(\xi)(x - y) \) for some \( \xi \) between \( x, y \). Also, \( F \) is a contractive map in a closed set \( C \) if there exists \( \lambda < 1 \) such that \( |F(x) - F(y)| \leq \lambda |x - y| \) for all \( x, y \in C \).