Homework #3

1. (b) Plugging in data points, we get the system of linear equations:

   \[ a = 2 \]
   \[ b = 3 \]
   \[ c = -2. \]

   So

   \[ p(x) = \frac{1}{3}(x-1)(x-2) - \frac{3}{2}(x+1)(x-2) - \frac{2}{3}(x+1)(x-1). \]

   (c) Plugging in data points, we get the system of linear equations:

   \[ a = 2 \]
   \[ a + 2b = 3 \]
   \[ a + 3b + 3c = -2. \]

   Solving from top to bottom, \( a = 2 \) and \( b = (3 - 2)/2 = 1/2 \) and \( c = (-2 - 2 - 3/2)/3 = -11/6. \) So

   \[ p(x) = 2 + \frac{1}{2}(x+1) - \frac{11}{6}(x+1)(x-1). \]

3. (a) The Lagrange form of the Lagrange polynomial takes the form:

   \[ p(x) = f(x_0 - h) \frac{(x-x_0)(x-x_0-h)}{(-h)(-2h)} + f(x_0) \frac{(x-x_0+h)(x-x_0-h)}{(h)(-h)} + f(x_0 + h) \frac{(x-x_0+h)(x-x_0)}{2h(h)} \]

   \[ = f(x_0 - h) \frac{(x-x_0)(x-x_0-h)}{2h^2} - f(x_0) \frac{(x-x_0+h)(x-x_0-h)}{h^2} + f(x_0 + h) \frac{(x-x_0+h)(x-x_0)}{2h^2}. \]

   (b) Taking a derivative, plugging in \( x_0 \), and simplifying:

   \[ p'(x_0) = f(x_0 - h) \frac{-h}{2h^2} - f(x_0) \frac{-h + h}{h^2} + f(x_0 + h) \frac{h}{2h^2} \]

   \[ = -\frac{f(x_0 - h)}{2h} + \frac{f(x_0 + h)}{2h} \]

   \[ = \frac{f(x_0 + h) - f(x_0 - h)}{2h}. \]
4. (a) The divided difference table looks like:

\[
\begin{array}{ccc}
-1 & -2 & 2 \\
0 & 0 & 1/2 \\
1 & 3 & \\
\end{array}
\]

(b) iii. The Newton form for the Lagrange polynomial is:

\[p(x) = -2 + 2(x + 1) + \frac{1}{2}(x + 1)x.\]

6. (Matlab)

(a) See “hw4afn.m”.

(b) The function gives an approximation of 1.57222222222222 for \(f(2)\).

9. (Math 274) Given \(p, q\) two polynomials of degree \(\leq 3\) interpolating the data, let \(r = p - q\), a polynomial of degree \(\leq 3\). Then \(r' = p' - q'\), a polynomial of degree \(\leq 2\). From the data, \(r'(x_i) = p'(x_i) - q'(x_i) = z_i - z_i = 0\), for \(i = 0, 1\), so, since \(x_0 \neq x_1\), \(r'\) has at least 2 roots. In addition, \(r(x_i) = p(x_i) - q(x_i) = y_i - y_i = 0\), for \(i = 0, 1\), so \(r\) has roots at \(x_0, x_1\). Thus, by Rolle’s Theorem, \(r'\) has a root at some \(\xi\) between \(x_0, x_1\). So \(r'\) has at least three roots: \(x_0, x_1, \xi\). But this is more than its degree, so Fundamental Theorem of Algebra implies \(r' \equiv 0\). Integrating, \(r\) is a constant function, and, since it has roots at the nodes, \(r \equiv 0\) and \(p \equiv q\).